Resúmenes de conferencias

MV-algebras beyond algebraic logic

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MV-algebras stand to boolean algebras as the Łukasiewicz infinite-valued calculus stands to boolean propositional logic, [1]. We will discuss our recent work [2]-[8] on MV-algebras, jointly with Busaniche and Cabrer, with the aim of showing that the scope of MV-algebra theory goes far beyond algebraic logic.

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What is logical truth?

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We use metamathematical tools to compare and evaluate two familiar, but strikingly different definitions of the notion "logical truth", one from Hilbert and quantifying over sentences and their substitutions, and one from Russell, quantifying over truth-values.

On the basis of a set-theoretic exploration of logical truth both classical and intuitionistic, we show that, without the law of the excluded third, the two definitions are mathematically independent.

Syntactical reasons

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It is a long tradition in Universal Algebra and Model Theory to look for syntactical objetcs underlying structural behaviour. Birkhoff's celebrated HSP-Theorem is a premier example, and arguably the result that started it all. It says that a class $\mathscr{V}$ of algebraic structures is closed under the the formation of direct products, subalgebras and homomorphic images if and only if it can be axiomatized by *identities*, that is, $\mathscr{V}$ can be defined with properties of the form $\forall x_1,\ldots,x_n\, p(\bar{x}) = q(\bar{x})$. The utility and power of this result (witnessed by its countless applications) comes from the fact that it allows us to replace an abstract set of closure conditions for a concrete object (the set of identites), which can be formally manipulated and made part of our reasonings.

Following Birkhoff's seminal theorem many other discoveries explaining semantical properties through the existence of syntactical objects have been made (e.g., Malcev conditions, Congruence formulas, etc.). The belief that this kind of results provide both a deeper understanding of structural properties, and the potential of interesting applications, has been the drive behind the lines of work pursued by our research group at FaMAF in Córdoba. Led by Professor Vaggione, we have worked on several problems of the aforementioned nature. In our talk we will survey the results we have obtained and explain the methods and tools employed. We shall also discuss ongoing and future work.

Implication zroupoids: an abstraction from De Morgan algebras

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In 1934, Bernstein gave a system of axioms for Boolean algebras in terms of implication only; however, his original axioms were not equational. A quick look at his axioms would reveal that, with an additional constant, they could easily be translated into equational ones. In 2012, I extended this modified Bernstein's theorem to De Morgan algebras in [San12]. Indeed, I proved in [San12] that the varieties of De Morgan algebras, Kleene algebras, and Boolean algebras are term-equivalent, to varieties whose defining axioms use only the implication $\to$ and the constant 0.

These results motivated me to introduce a new (equational) class of algebras called "Implication zroupoids" in [San12].

An algebra $\mathbf{A} = \langle A, \to, 0\rangle$, where $\to$ is binary and 0 is a constant, and $x' := x \to 0$, is called an *implication zroupoid* ($\mathscr{I}$-zroupoid, for short) if $\mathbf{A}$ satisfies:

(I) $(x \to y) \to z \approx [(z' \to x) \to (y \to z)']'$,

(I_0) $0'' \approx 0$.

During the last two years, Juan Cornejo and I have continued these investigations into the stucture of the lattice of subvarieties of the variety of implication zroupoids in [CS16], [CS16a], [CS16b], [CS16c], [CS16d], [CS16e], [CS16f],and [CS16g].

In this talk I would like to survey some of our results on implication zroupoids and mention some new directions for future research.

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An algebraic modal logic view on subordination and contact algebras

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Precontact relations (a.k.a. proximity relations) on a Boolean algebra are the algebraic abstraction of the adjacency spaces of Düntch and Vakarelov. These spaces are used in the region-based theory of space. Contact relations are the precontact relations that are related to the proximity spaces of Efremovic.

Other relations on Boolean algebras considered in the literature are the subordination relations which turn to be equivalent to the precontact relations; both are also equivalent to the quasi-modal operators on Boolean algebras introduced by C. Celani. Recently all these relations have been studied in the context of the de Vries duality between the category of de Vries algebras and the category of compact Hausdorff spaces by authors like G. and N. Bezhanishvili and collaborators. In the talk, which surveys recent work in collaboration with S. Celani, we address the topic from a purely algebraic modal logic point of view.

In the first part of the talk, we will present several varieties of Boolean algebras with a binary modal operator that in a sense correspond to classes of Boolean algebras with a precontact relation. The basic varieties we introduce are the variety of pseudo-subordination algebras and its subvariety PSC of pseudo-contact algebras. It turns out that the contact Boolean algebras with a contact relation correspond exactly to the simple elements of PSC that satisfy an additional equational condition. We will present an axiomatization of the variety that these simple algebras generate.

In the second part of the talk we will discuss the dual topological spaces that correspond to the algebras we introduce in the first part using the standard tools of the topological duality for Boolean algebras with operators, and we will derive the known topological duality for precontact Boolean algebras.

Logic and implication

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Algebraic logic has developed a rich theory of non-classical propositional logics. Virtually all prominent logical systems studied in the literature have a reasonable notion of implication (that is, satisfying the following minimal requirements: reflexivity, transitivity, modus ponens, and symmetrized congruence). This has motivated the introduction of weakly p-implicational logics as an alternative presentation of protoalgebraic logics that highlight and exploit the role of implications. This talk will present such theory (joint work with P. Cintula, AML 2010-2017), starting with its motivations and basic definitions, and covering its main results and applications to the fields of substructural and semilinear logics.

Lattice-valued predicate logics

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Classical predicate logic interprets n-ary predicates as mappings from the n-th power of a given domain into the two-valued Boolean algebra 2.

The idea of replacing 2 by a more general structure is very natural and was shown to lead to very interesting results: prime examples are the Boolean-valued or Heyting-valued models of set theory (or even more general models proposed e.g. by Takeuti, Titani, Hajek and Hanikova).

There exists a stream of research, by Mostowski, Rasiowa, Sikorski, Horn, and Hajek to give just a few names, studying logics where predicates can take values in lattices (with additional operators) from a certain class. In this talk I present a general framework (joint work with C. Noguera, JSL 2015) for studying predicate logics where the mentioned class of lattices is the equivalent algebraic semantics of a propositional logic algebraizable in the sense of Blok and Pigozzi. I will also present some recent results (joint work with Metcalfe and Diaconescu, LPAR 2013 and LPAR 2015) on Skolem and Herbrand theorems for some of these logics.

Interpolation and Robinson's lemma in Łukasiewicz predicate logic

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It is well known that Craig interpolation property fails in Łukasiewicz propositional logic, but so called deductive interpolation holds [2]. Moreover, Craig interpolation holds for Łukasiewicz propositional logic enriched with division connectives [1]. It is easily seen that the failure of Craig's property lifts to Łukasiewicz predicate logic. On the other hand, we develop enough model theory to show that Robinson's joint consistency lemma holds in Łukasiewicz predicate logic and obtain an approximate version of deductive interpolation in which the interpolant may be a countable theory. We discuss the possibility of having sharp interpolants as in the propositional fragment and the possibility that divisible Łukasiewicz predicate logic satisfies Craig interpolation.

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A categorical equivalence for distributive Stonean residuated lattices

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In this talk we present a categorical equivalence for the category $\mathscr{S}$ of distributive Stonean residuated lattices. These residuated lattices form a variety that can be characterized as the greatest subclass of bounded distributive residuated lattices that satisfy that the double negation is a retract onto its Boolean skeleton. It contains important subvarieties as the variety of Boolean algebras, Gödel algebras, product algebras and pseudocomplemented MTL-algebras. Objects of $\mathscr{S}$ are closely related to Stone algebras (also known as Stone lattices, see [2]), since the bounded lattice reduct $\mathbf{L}(\mathbf{A})$ of $\mathbf{A} \in \mathscr{S}$ is a Stone algebra.

The equivalence that we present, following some of the ideas of [1] and [3], is between the category $\mathscr{S}$ and a category $\mathscr{T}$ of triples. We define a funtor $\mathbf{T}$ from $\mathscr{S}$ into $\mathscr{T}$ that assigns to each object $\mathbf{A} \in \mathscr{S}$ a triple $(\mathbf{B}, \mathbf{D}, \phi)$ where $\mathbf{B}$ is the Boolean skeleton of $\mathbf{A}$, $\mathbf{D}$ is the algebra of dense elements of $\mathbf{A}$ and ϕ is connecting map that relates these two substructures. After showing that $\mathbf{T}$ defines an equivalence, we present some applications and compare our equivalence with the one in [4] for the subcategory of product algebras.

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* Based on a joint work with Roberto Cignoli and Miguel Andrés Marcos.

Admissible rules and (almost) structural completeness for many-valued logics

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Admissible rules of a logic are those rules under which the set of theorems are closed.

A logic is said to be structurally complete when every admissible rule is a derivable rule.

A logic is almost structurally complete when every rule is either derivable or passive (there is no substitution that turns all premisses into theorems).

Gödel logic is structurally complete and moreover every axiomatic extensión is structurally complete; finite-valued Lukasiewicz logics turn to be almost structurally complete and the infinite valued Lukasiewicz logic is not structurally complete nor almost structurally complete.

In this talk we will algebraically investigate structural completenss, almost structural completeness and bases of admisible rules for some cases of algebraizable many-valued logics: particularly Lukasiewicz logics and nilpotent minimum logics.

Comunicaciones de Álgebra

Cohomología parcial de grupos

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Las acciones parciales de grupos, que generalizan a las acciones de grupos, aparecen naturalmente en el estudio de álgebras generadas por isometrías de un espacio de Hilbert y en el estudio de las álgebras de caminos de Leavitt. Nuestro objetivo es introducir el concepto de cohomología parcial, generalizando el concepto de cohomología de grupos usual.

En el trabajo [Alvares, E.R.; Alves, M.; Redondo, M.J.: Cohomology of partial smash products. J. Algebra 482 (2017), 204–223] definimos la cohomología parcial de grupos como el funtor derivado a derecha del funtor de invariantes parciales, y mostramos que existe una sucesión espectral que relaciona la cohomología de productos smash parciales con la cohomología parcial de grupos y la cohomología de álgebras.

Primeros grupos de la cohomología de Hochschild de extensiones parciales por relaciones

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Sea C un álgebra triangular de dimensión global menor o igual a dos y sea $\tilde{C} = C \ltimes \mathrm{Ext}^2_C(DC,C)$ su extensión por relación. Supongamos que $E = \mathrm{Ext}^2_C(DC,C) = E' \oplus E''$ como $C-C$ bimódulos y consideremos la extensión parcial por relaciones $B = C \ltimes E'$. El propósito de esta charla será relacionar los primeros grupos de cohomología de Hochschild de las álgebras C, B y $\tilde{C}$ aplicando las técnicas de [AGST].

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Sobre la dimensión global fuerte de un álgebra

Claudia Chaio, Alfredo González Chaio, **Nilda Pratti**

Sea A un álgebra de dimensión finita sobre un cuerpo algebraicamente cerrado. Denotamos por $\operatorname{mod} A$ la categoría de los A-módulos finitamente generados a derecha y por $\operatorname{proy} A$ la subcategoría llena de $\operatorname{mod} A$ de los A-módulos proyectivos finitamente generados.

En [BSZ], los autores definieron y estudiaron las categorías $\mathbf{C_n}(\operatorname{proy} A)$ de complejos de módulos proyectivos finitamente generados concentrados en un intervalo finito. Estas categorías son de tipo de representación finito si tienen sólo un número finito de complejos indescomponibles, salvo isomorfismos.

Por otro lado, el concepto de dimensión global fuerte fue introducida por Skowronsky en [S]. Para un complejo $X \in \mathbf{K}^b(\text{proy A})$,

$$\cdots \to 0 \to 0 \to X^r \to X^{r+1} \to \cdots \to X^{s-1} \to X^s \to 0 \to 0 \cdots$$

con $X^r \neq 0$ y $X^s \neq 0$, se define la longitud de X como $\ell(X) = s - r$. La dimensión global fuerte de A, $s.gl.\dim A$, es el supremo del conjunto $\{\ell(X) \mid X \in K^b(proy\, A)$ es indescomponible$\}$.

En este trabajo mostraremos que si A un álgebra con $s.gl.dimA = \eta < \infty$ entonces, para todo $n \geq 2$, $\mathbf{C_n}(\text{proy A})$ es de tipo de representación finito si y sólo si $\mathbf{C}_{\eta+\mathbf{1}}(\text{proy A})$ es de tipo de representación finito.

También, consideraremos algunas algebras con dimensión global fuerte finita y mostraremos como calcular esta dimension a partir de su carcaj ordinario con relaciones. Más aún, mostraremos la forma de un complejo de longitud máxima en esos casos.

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Extensiones triviales de álgebras monomiales. Aplicación al caso gentil y su relación con las álgebras de grafo de Brauer

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Sea A un álgebra de dimensión finita sobre un cuerpo k algebraicamente cerrado. Supondremos A básica e indescomponible, es decir, $A = kQ/I$ donde Q es un quiver finito y conexo e I es un ideal de relaciones. La extensión trivial $T(A) = A \ltimes D(A)$ de A por $D(A)$ es el álgebra cuyo k - espacio vectorial subyacente es $A \times D(A)$, con el producto dado por: $(a, f)(b, g) = (ab, ag + fb)$ para $a, b \in A$ $f, g \in D(A)$.

Fernández y Platzeck describieron en [FP] el quiver de la extensión trivial de un álgebra de dimensión finita A. También dieron las relaciones de dicha extensión trivial cuando el quiver de A no tiene ciclos orientados no nulos.

En esta comunicación describiremos las relaciones de la extensión trivial de un álgebra monomial. Aplicaremos este resultado al caso particular en que A es un álgebra gentil, y veremos que las extensiones triviales de las mismas coinciden con las álgebras de grafo de Brauer con multiplicidad uno en todos sus vértices.

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Una revisión de la inversa core EP

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La inversa core EP fue introducida por Prasad y Mohana en [4] para matrices cuadradas de índice arbitrario generalizando el concepto de inversa core estudiada en [1] para matrices cuadradas de índice 1. El objetivo de esta propuesta es realizar una revisión de la inversa core EP. Por un lado se da una nueva caracterización y representación de dicha inversa usando la descomposición core EP obtenida recientemente en [5]. Por otro lado, a partir de esta nueva caracterización se observan algunas propiedades similares a las que cumplen las inversas BT [2] y DMP [3], que son también otras generalizaciones de la inversa core. Finalmente se obtiene una forma canónica de la inversa core EP a partir de la descomposición de Hartwig-Spindelböck que provee una manera sencilla de calcularla.

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The spectra of arrangement graphs

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Arrangement graphs were introduced for their connection to computational networks and have since generated considerable interest in the literature. In two recent articles [2] and [3], the eigenvalues of the adjacency matrix are studied. The main result in [3] is that the eigenvalues are integers. In this article we study the adjacency matrix from the perspective of the representation theory of symmetric groups. In particular, we consider the representation associated to the arrangement graph and the corresponding equivariant operator associated to the adjacency matrix. Our approach leads to a simple derivation for an explicit formula of the spectra of (n,k)-arrangement graphs in terms of the characters of irreducible representations evaluated on a transposition.

As an application of our formula, we prove a conjecture by Chen, Ghorbani and Wong that states for k fixed and n large, $-k$ is the only negative eigenvalue in the spectrum of the (n,k)-arrangement graph.

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Sobre el espectro de grafos de arreglos

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Para $r \leq k \leq n$ números enteros no negativos, $A(n,k,r)$ es el grafo cuyo conjunto de vértices son todas las k-permutaciones de un conjunto con n elementos, y dos vértices son adyacentes si ellos difieren en exactamente r posiciones. Los grafos $A(n,k,r)$ pueden interpretarse como una extensión de los grafos de arreglos $A(n,k)$ introducidos en [5] en conexión con computación paralela.

Chen, Ghorbani y Wong estudian en [3] el espectro de los grafos $A(n,k)$ para valores chicos de k, establecen que el espectro del grafo de Johnson $J(n,k)$ forma parte del espectro de $A(n,k)$ y determinan los valores extremos en el espectro de $A(n,k)$. Los mismos autores en [4] prueban que el espectro del grafo de arreglos $A(n,k)$ es entero.

En [1] Araujo y Bratten obtienen el espectro del grafo de Johnson $J(n,k,r)$ utilizando la teoría de representaciones del grupo simétrico, y en [2] utilizando técnicas análogas determinan el espectro del grafo de arreglos $A(n,k)$, el cuál se describe en término de los valores alcanzados sobre las transposiciones por caracteres irreducibles de grupos simétricos y de los los grados de estos caracteres.

Sobre la base de lo realizado en [2], estudiamos el espectro del grafo $A(n,k,2)$. En esta comunicación presentamos una fórmula explícita para obtener el espectro del grafo $A(n,k,2)$. A partir de la misma se deduce que este grafo no tiene espectro entero, siendo el mismo un subconjunto de $\frac{1}{2}\mathbb{Z}$.

El resultado obtenido se resume en el siguiente teorema.

Teorema: Sean τ_n, τ_k transposiciones en $\mathfrak{S}_n$ y $\mathfrak{S}_k$ respectivamente y sean σ_n, σ_k triciclos en $\mathfrak{S}_n$ y $\mathfrak{S}_k$ respectivamente. Entonces el espectro del grafo $A(n,k,2)$ está dado por

$$\begin{aligned} &\frac{1}{2}\left[\binom{n}{2}\frac{\chi_\mu(\tau_n)}{\chi_\mu(1)} - \binom{k}{2}\frac{\chi_\lambda(\tau_k)}{\chi_\lambda(1)} - \binom{n-k}{2}\right]^2 \\ &\qquad - (n-k-1)\left[\binom{n}{2}\frac{\chi_\mu(\tau_n)}{\chi_\mu(1)} - \binom{k}{2}\frac{\chi_\lambda(\tau_k)}{\chi_\lambda(1)} - \binom{n-k}{2}\right] \\ &\qquad + \binom{n}{2}\frac{\chi_\mu(\sigma_k)}{\chi_\mu(1)} + \binom{k}{2}\frac{\chi_\lambda(\tau_n)}{\chi_\lambda(1)} - \binom{k}{3}\frac{\chi_\lambda(\sigma_n)}{\chi_\lambda(1)}I - \binom{n-k}{3}I - \frac{k(n-k)}{2} \end{aligned}$$

donde λ es una partición de k, μ es una partición de n tales que $\mu_i' \leq \lambda_i' + 1$, siendo μ' y λ' las particiones conjugadas de μ y λ respectivamente.

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Comunicaciones de Análisis

Estimaciones con pesos para extensiones vectoriales

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Dado un peso $w \in A_p$, un resultado que ha causado mucho impacto en el Análisis Armónico ha sido la obtención de la norma de un operador T en L^p_w en términos de la constante $[w]_{A_p}$. Esto fue estudiado para distintos operadores tales como la maximal de Hardy-Littlewood, los operadores de Calderón-Zygmund y sus conmutadores. Estos resultados se han mejorado en términos de la constante $[w]_{A_\infty}$. Estas desigualdades se conocen como resultados mixtos de tipo $A_p - A_\infty$.

En esta charla mencionaremos las generalizaciones que obtuvimos en esta línea para las extensiones vectoriales de la maximal de Hardy-Littlewood y los operadores de Calderón -Zygmund. Estas estimaciones están basadas en ciertas estimaciones puntuales de estos operadores en términos de operadores de tipo "sparse". Por otro lado daremos resultados obtenidos para el caso mixto $A_p - A_\infty$ también en el caso vectorial.

Desigualdades con diferentes pesos para operadores multilineales

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En este trabajo estudiamos desigualdades mixtas con pesos para operadores multilineales y (sub)multilineales.

Vale el siguiente teorema: Sean $w_1, \ldots, w_m \in A_1$ y sea $v \in A_\infty$. Sea $\nu = w_1^{\frac{1}{m}} \ldots w_m^{\frac{1}{m}}$. Entonces existe una constante C tal que

$$\left\| \frac{\prod_{i=1}^m Mf_i}{v} \right\|_{L^{\frac{1}{m},\infty}(\nu v^{\frac{1}{m}})} \leq C \prod_{i=1}^m \|f_i\|_{L^1(w_i)}.$$

Este teorema generaliza al contexto multilineal el resultado de Sawyer sobre desigualdades mixtas. Ver [4] y [4]. Además observar que es el caso más singular, ya que $v \in A_\infty$. Ver [3].

Como corolario de este teorema y teniendo en cuenta la definición del operador (sub) multilineal $\mathscr{M}$ definido en [2]

$$\mathscr{M}(\vec{f})(x) = \sup_{x \in Q} \prod_{i=1}^m \frac{1}{|Q|} \int_Q |f_i(y_i)| dy_i,$$

donde $\vec{f} = (f_1, \ldots, f_m)$ y el supremo es tomado sobre todos los cubos Q que contienen a x, tenemos que vale: Sean $w_1, \ldots, w_m \in A_1$ y sea $v \in A_\infty$. Sea $\nu = w_1^{\frac{1}{m}} \ldots w_m^{\frac{1}{m}}$. Entonces existe una constante C tal que

$$\left\| \frac{\mathscr{M}(\vec{f})(x)}{v} \right\|_{L^{\frac{1}{m},\infty}(\nu v^{\frac{1}{m}})} \leq C \prod_{i=1}^m \|f_i\|_{L^1(w_i)}.$$

A partir de este último corolario y argumentos de extrapolación podemos extender nuestro resultado a operadores de Calderón-Zygmund multilineales.

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Factorización del operador de marco y localización de marcos en espacios de Hilbert

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Karlheinz Gröchenig en sus trabajos sobre localización de marcos en espacios de Hilbert publicados en 2003 y 2004 introduce una definición para *localización de un marco* respecto de una dada base de Riesz para un espacio de Hilbert separable $\mathbf{H}$. En este contexto prueba que si un marco $\mathscr{E} = (e_x)_{x\in\mathscr{X}}$ para $\mathbf{H}$, donde $\mathscr{X}$ es un subconjunto de $\mathbb{R}^d$ con ciertas propiedades entre ellas la numerabilidad, tiene localización polinomial o exponencial entonces su marco dual canónico tiene la misma localización. Para probar este resultado, obtiene una factorización del operador de marco $S_{\mathscr{E}}$ asociado a $\mathscr{E} = (e_x)_{x\in\mathscr{X}}$ en términos de ciertos operadores relacionados con la base de Riesz dada y una matriz infinita relacionada con el marco dado. Puesto que el operador de marco asociado al marco dual canónico $(S_{\mathscr{E}}^{-1}e_x)_{x\in\mathscr{X}}$, es $S_{\mathscr{E}}^{-1}$ mediante el Teorema de Jaffard prueba que la matriz infinita asociada a $S_{\mathscr{E}}^{-1}$ es la matriz inversa de la matriz infinita encontrada en la factorización del operador de marco original.

En el presente trabajo se estudia el siguiente problema: Dado un marco $\mathscr{E} = (e_x)_{x\in\mathscr{X}}$ o una sucesión de marco para un espacio de Hilbert $\mathbf{H}$, con localización polinomial o exponencial, y un operador $A \in \mathfrak{B}(\mathbf{H})$ de rango cerrado, bajo qué condiciones $A(\mathscr{E})$ tiene localización, si esta es o no la misma que la del marco o sucesión de marco dados, respecto de la misma base de Riesz dada u otra base de Riesz.

Se obtienen factorizaciones del operador de marco asociado a $A(\mathscr{E})$ en términos del operador de marco original, $S_{\mathscr{E}}$, y una matriz infinita que depende del marco, la base de Riesz dada y otra base de Riesz elegida adecuadamente, en cada caso. La factorización obtenida para el operador de marco asociado a $A(\mathscr{E})$ constituye una generalización adecuada para el Teorema de Jaffard. Esta factorización permite introducir la localización original del marco $\mathscr{E}$ en la factorización del operador de marco asociado a $A(\mathscr{E})$ y así poder obtener los resultados buscados. Los resultados arriba descriptos se aplican al estudio de la localización de $A(\mathscr{E})$ en términos de la localización de $\mathscr{E}$, del operador A, y de las bases de Riesz que surgen del proceso de estudio de la localización, o de la base de Riesz dada. Se enuncian condiciones necesarias y/o suficientes sobre el operador A y el marco dado para la localización de $A(\mathscr{E})$.

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Sobre propiedades de conexión en espacios topológicos

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Estudiamos la estructura abstracta de las propiedades de conexión en espacios topológicos arbitrarios, atendiendo a cómo se comportan en todos los subespacios. También se analiza la relación de las propiedades de conexión con respecto a otras propiedades topológicas, como la compacidad y varias propiedades de separación. Prestamos particular atención a la estructura de los conjuntos conexos del sistema de los números reales.

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The weak*-closure of certain subspaces of the dual of some abelian Banach algebras

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We shall determine the weak*-closure of some subspaces of the dual of certain Banach algebras. This matter will be related with the problematic concerning to the existence of bounded approximate identities within closed ideals of abelian semisimple Banach algebras provided with bounded approximate identities.

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Comunicaciones de Probabilidad y Estadística

Vitamin D level statistics analisys for Argentina's central region

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Many recent research has shown that a large number of processes in the human body involve vitamin D (VD), which indicates its importance. Although there are international intake recommendations for normal bone development, there is currently no agreement regarding adequate VD levels for the rest of its metabolic roles.

As VD it is mainly synthesized in the skin by sun exposure we explore, in previous works, how these levels evolve during a year and geographically with the latitude.

Working with data from all over the country, we divided the territory into four zones, and the statistical tests found highly significant evidence of difference in the proportion of people with healthy levels of VD by both season and geographical region.

Besides that we develop time series techniques, including cross correlation calculations, that gave strong delayed correlations with daily sunlight hours amount, and allowed us to estimate that the maximum correlation is reached at lag of forty three days.

In this work we will analyze more specifically the data from the central regions, to compare the results for alternative VD levels. We use non-parametric correlation calculation to estimate delay with the daily amount of hours of sunlight when the zones with extreme variation data are removed.

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Comunicaciones de Geometría

Geometría de los sistemas hamiltonianos con puertos

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En esta chala recordaremos brevemente la noción de sistema hamiltoniano con puertos. A continuación, utilizando un formalismo basado en las estucturas de Dirac, intentaremos poner de manifiesto la geometría de estos sistemas y describir la noción de interconexión.

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Hamiltonización en teorías de campo y triples de Tulczyjew

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El problema inverso (tanto en Mecánica como teoría de campos) intenta caracterizar aquellas ecuaciones diferenciales (ordinarias y parciales, respectivamente) que son las ecuaciones de Euler-Lagrange asociadas a algún problema variacional [11, 12]. El problema análogo para las ecuaciones de Hamilton se conoce con el nombre de *Hamiltonización* [9, 4, 10, 8, 1]: Aquí se intenta hallar condiciones que aseguren que un determinado sistema de ecuaciones diferenciales sea equivalente a las *ecuaciones de Hamilton-DeDonder-Weyl* [3, 5, 6, 7] correspondientes a una determinada densidad hamiltoniana.

En la presente comunicación utilizaremos una formulación de teorías de campo de primer orden mediante triples de Tulczyjew [2] para intentar resolver el problema de hamiltonización en tales teorías.

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La grassmanniana restringida y el teorema de foliación simpléctica

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En esta exposición, presentaremos el principal resultado de [2]: la grassmanniana restringida es una hoja simpléctica en un espacio de Banach-Lie Poisson de distribución característica integrable. Para dar este resultado, repasaremos la noción de variedades de Poisson modeladas en espacio de Banach introducidas por A. Odzijewicz y T. Ratiu [3]. En este contexto, la prueba del teorema clásico de foliación simpléctica para variedades finito dimensionales no se puede reproducir. Así, resulta interesante conocer ejemplos como el mencionado relativo a la grassmanniana restringida. Esta grassmanniana es un espacio homogéneo de dimensión infinita que desempeña un papel importante en diversas áreas de matemática y física. Está vinculada con los grupos de lazos [4], la integrabilidad de ecuaciones del tipo KdV [5]. Su estructura de variedad de Khäler fue estudiada en [6] y como variedad Riemanniana de dimensión infinita, el teorema de Hopf-Rinow fue demostrado en [1]. Cabe aclarar que los resutlados a exponer no son originales, y forman parte de un trabajo de iniciación a la investigación.

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Los campos de Jacobi del grupo de Lie de las transformaciones de Moebius de la circunferencia

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El comportamiento de las geodésicas de una variedad riemanniana puede ser estudiado mediante los campos de Jacobi. Los campos de Jacobi resultan ser una poderosa herramienta para estudiar la geometría intrínseca y extrínseca de una variedad riemanniana. Los *campos de Jacobi* son campos vectoriales definidos a lo largo de una curva geodésica de una variedad riemanniana que satisface una ecuación diferencial de segundo orden, involucra al operador de curvatura y está asociado a una variación de geodésicas.

En este charla, mostraremos cómo utilizando esta herramienta podemos describir el comportamiento de las geodésicas del grupo de Lie $\mathscr{M}$ de las transformaciones de Moebius de la circunferencia, que resulta una variedad con curvatura variable.

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Comunicaciones de Lógica

On the poset product representation of BL-algebras

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The variety $\mathscr{BL}$ of BL-algebras is the algebraic counterpart of BL, the logic presented by Hájek which includes a fragment common to the most important fuzzy logics (Łukasiewicz, Gödel and product logics). Likewise any variety, $\mathscr{BL}$ is generated by its totally ordered members, namely BL-chains. Since every BL-algebra can be embedded into the direct product of BL-chains and every BL-chain can be decomposed as an ordinal sum of simpler structures, Jipsen and Montagna proposed in [3] a construction called poset product as a sort of generalization of direct product and ordinal sum.

In [2], based on the results of [4], it is shown that every BL-algebra is a subalgebra of a poset product of MV-chains and product chains with respect to a poset which is a forest. Although this embedding provides a representation for finite BL-algebras, some limitations arise from the infinite case. Moreover, in [1] there are easy examples of BL-chains are not representable in the sense of poset product.

The aim of this communication is to examine some features of the poset product construction in the context of $\mathscr{BL}$. We will first consider the restrictions referred above in order to introduce the notion of idempotent free BL-chain. Then, we will suggest some requirements that a BL-algebra should satisfy so that it admit a representation as a poset product of idempotent free BL-chains.

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Forest products of MTL-chains

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A *semihoop* is an algebra $\mathbf{A} = (A, \cdot, \to, \wedge, \vee, 1)$ of type $(2,2,2,2,0)$ such that $(A, \wedge, \vee)$ is lattice with 1 as greatest element, $(A, \cdot, 1)$ is a commutative monoid and for every $x, y, z \in A$: *(i)* $xy \leq z$ if and only if $x \leq y \to z$, and *(ii)* $(x \to y) \vee (y \to x) = 1$. A semihoop $\mathbf{A}$ is *bounded* if $(A, \wedge, \vee, 1)$ has a least element 0. A *MTL-algebra* is a bounded semihoop. Hence, MTL-algebras are prelinear integral bounded commutative residuated lattices (c.f. [2]). A MTL-algebra $\mathbf{A}$ is a *MTL chain* if its semihoop reduct is totally ordered. A *forest* is a poset X such that for every $a \in X$ the set

$$\downarrow a = \{x \in X \mid x \leq a\}$$

is a totally ordered subset of X.

Inspired by the concept of poset products of BL-algebras (c.f. [1]) we define the *forest product* of MTL-chains in this manner:

Definition 1. Let $\mathbf{F} = (F, \leq)$ a forest and let $\{\mathbf{M}_i\}_{i\in\mathbf{F}}$ a collection of MTL-chains such that, up to isomorphism, all they share the same neutral element 1. If $(\bigcup_{i\in\mathbf{F}})^F$ denotes the set of functions $h : F \to \bigcup_{i\in\mathbf{F}} \mathbf{M}_i$ such that $h(i) \in \mathbf{M}_i$ for all $i \in \mathbf{F}$, the forest product $\bigotimes_{i\in\mathbf{F}} A_i$ is the algebra $\mathbf{M}$ defined as follows:

(1) The elements of $\mathbf{M}$ are the $h \in (\bigcup_{i\in\mathbf{F}})^F$ such that, for all $i \in \mathbf{F}$ if $h(i) \neq 1$ then for all $j > i$, $h(j) = 0$.
(2) The monoid operation and the lattice operations are defined pointwise.
(3) The residual is defined as follows:

$$(h \to g)(i) = \begin{cases} h(i) \to_i g(i), & \text{if for all } j < i,\ h(j) \leq_j g(j) \\ 0 & \text{otherwise} \end{cases}$$

where the subscript $_i$ denotes the realization of operations and of order in $\mathbf{M}_i$.

In every forest $\mathbf{F}$ the collection $\mathscr{D}(\mathbf{F})$ of lower sets of $\mathbf{F}$ defines a topology over F called the *Alexandrov topology* on $\mathbf{F}$. The purpose of this communication is to study the forest product of MTL-chains in order to prove the following statement: *The forest product of MTL-chains is essentially a sheaf of MTL-algebras over an Alexandrov space whose fibers are MTL-chains.*

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A representation for the n-generated free algebra in the subvariety of BL-algebras generated by $[0,1]_{\mathbf{MV}} \oplus [0,1]_{\mathbf{G}}$

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BL-algebras were introduced by Hájek (see [1]) to formalize fuzzy logics in which the conjunction is interpreted by continuous t-norms over the real interval $[0,1]$. These algebras form a variety, usually called $\mathscr{BL}$. Important examples of its proper subvarieties are the variety $\mathscr{MV}$ of MV-algebras, $\mathscr{P}$ of product algebras and $\mathscr{G}$ of Gödel algebras.

For each integer $n \geq 0$, we will write $Free_{\mathscr{BL}}(n)$ to refer to the free n-generated BL-algebra, which is generated by the algebra $(n+1)[0,1]_{\mathbf{MV}}$, that is, the ordinal sum of $n+1$ copies of the standard MV-algebra. This fact allows us to characterize the free n-generated BL-algebra $Free_{\mathscr{BL}}(n)$ as the algebra of functions $f : (n+1)[0,1]^n_{\mathbf{MV}} \to (n+1)[0,1]_{\mathbf{MV}}$ generated by the projections. Using this, in [3] and [2] there is a representation of the free-n-generated BL-algebra in terms of elements of free Wajsberg hoops ($\perp$-free subreducts of Wajsberg algebras), organized in a structure based on the ordered partitions of the set of generators and satisfying certain geometrical constraints.

In this work we will concentrate in the subvariety $\mathscr{V} \subseteq \mathscr{BL}$ generated by the ordinal sum of the algebra $[0,1]_{\mathbf{MV}}$ and the Gödel hoop $[0,1]_{\mathbf{G}}$, that is, generated by $\mathbf{A} = [0,1]_{\mathbf{MV}} \oplus [0,1]_{\mathbf{G}}$. Though it is well-known that $[0,1]_{\mathbf{G}}$ is decomposable as an infinite ordinal sum of two-elements Boolean algebra, the idea is to treat it as a whole block. The elements of this block are the dense elements of the generating chain and the elements in $[0,1]_{\mathbf{MV}}$ are usually called regular elements of $\mathbf{A}$. The main advantage of this approach, is that unlike the work done in [3] and [2], when the number n of generators of the free algebra increase the generating chain remains fixed. This provides a clear insight of the role of the two main blocks of the generating chain in the description of the functions in the free algebra: the role of the regular elements and the role of the dense elements.

We give a functional representation for the free algebra $Free_{\mathscr{V}}(n)$. To define the functions in this representation we need to decompose the domain $[0,1]_{\mathbf{MV}} \oplus [0,1]_{\mathbf{G}}$ in a finite number of pieces. In each piece a function $F \in Free_{\mathscr{V}}(n)$ coincides either with McNaughton functions or functions on the free algebra in the variety of Gödel hoops (which we define using a base different from the one given by Gerla in [4]) in the following way:

- For every $\bar{x} \in ([0,1]_{\mathbf{MV}})^n$, $F(\bar{x}) = f(\bar{x})$, where f is a function of $Free_{\mathscr{MV}}(n)$.

For the rest of the domain, the functions depend on this function $f : ([0,1]_{\mathbf{MV}})^n \to [0,1]_{\mathbf{MV}}$:

- On $([0,1]_{\mathbf{G}})^n$: If $f(\bar{1}) = 0$, then $F(\bar{x}) = 0$ for every $\bar{x} \in ([0,1]_{\mathbf{G}})^n$, and if $f(\bar{1}) = 1$, then $F(\bar{x}) = g(\bar{x})$, for a function $g \in Free_{\mathscr{G}}(n)$, for every $\bar{x} \in ([0,1]_{\mathbf{G}})^n$.

Let $B = \{x_{\sigma(1)}, \dots x_{\sigma(m)}\}$ be a non empty proper subset of the set of variables $\{x_1, \dots, x_n\}$ and R_B be the subset of $([0,1]_{MV} \oplus [0,1]_G)^n$ where $x_i \in B$ if and only if $x_i \in [0,1]_G$. For every $\bar{x} \in R_B$ we also define $\tilde{x}$ as:

$$\tilde{x}_i = \begin{cases} x_i & \text{if } x_i \notin B \\ 1 & \text{if } x_i \in B \end{cases}$$

- On R_B: If $f(\tilde{x}) < 1$ then $F(\bar{x}) = f(\tilde{x})$, and if $f(\tilde{x}) = 1$, then there is a regular triangulation Δ of $f^{-1}(1) \wedge R_B$ which determines the simplices $S_1, \dots, S_n$ and l Gödel functions $h_1, \dots, h_n$ in $n-m$ variables $x_{\sigma(m+1)}, \dots, x_{\sigma(n)}$ such that
$F(\bar{x}) = h_i(x_{\sigma(m+1)}, \dots, x_{\sigma(n)})$ for each point $(x_{\sigma(1)}, \dots x_{\sigma(m)})$ in the interior of S_i.

This representation allows us to give a simple characterization of the maximal filters in this free algebra.

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States of free product algebras

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Within the framework of t-norm based fuzzy logics as developed by Hájek [2], *states* were first introduced by Mundici [3] as maps averaging the truth-value in Łukasiewicz logic. Such functions on MV-algebras (the algebraic counterpart of Łukasiewicz logic), suitably generalize the classical notion of finitely additive probability measures on Boolean algebras, besides corresponding to convex combinations of valuations in Łukasiewicz propositional logic. One of the most important results of MV-algebraic state theory is Kroupa-Panti theorem [4, §10], showing that every state of an MV-algebra is the Lebesgue integral with respect to a regular Borel probability measure. Many attempts of defining states in different structures have been made. In particular, in [1], the authors provide a definition of state for the Lindenbaum algebra of Gödel logic that results in corresponding to the integration of the truth value functions induced by Gödel formulas, with respect to Borel probability measures on the real unit cube $[0,1]^n$. Such states correspond to convex combinations of finitely many truth-value assignments.

The aim of this contribution is to introduce and study states for the Lindenbaum algebra of product logic, the remaining fundamental t-norm fuzzy logic for which such a notion is still lacking. Denoting with $\mathscr{F}_{\mathbb{P}}(n)$ the free n-generated product algebra, that is isomorphic to the Lindenbaum algebra of product logic formulas built from n propositional variables, a *state* will be a map from $\mathscr{F}_{\mathbb{P}}(n)$ to $[0,1]$ satisfying the following conditions: (i) $s(1)=1$ and $s(0)=0$; (ii) $s(f\wedge g)+s(f\vee g)=s(f)+s(g)$; (iii) If $f\leq g$, then $s(f)\leq s(g)$; (iv) If $f\neq 0$, then $s(f)=0$ implies $s(\neg\neg f)=0$.

Our main result is an integral representation for states of free product algebras. More precisely, a $[0,1]$-valued map s on $\mathscr{F}_{\mathbb{P}}(n)$ is a state in our sense if and only if there is a unique regular Borel probability measure μ such that, for every $f\in\mathscr{F}_{\mathbb{P}}(n)$,

$$s(f)=\int_{[0,1]^n} f\,\mathrm{d}\mu.$$

Moreover, we prove that every state belongs to the convex closure of product logic valuations. Indeed, in particular, extremal states will result in corresponding to the homomorphisms of $\mathscr{F}_{\mathbb{P}}(n)$ into $[0,1]$, that is to say, to the valuations of the logic.

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NPc-algebras and Gödel hoops

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This is a joint work with S. Aguzzoli, M. Busaniche and B. Gerla. See [2].

The algebraic models of paraconsistent Nelson logic were introduced by Odinstov under the name of N4-lattices.

NPc-lattices are a variety of residuated lattices that turn out to be termwise equivalent to eN4-lattices, the expansions of N4-lattices by a constant e. Thus paraconsistent Nelson logic can be studied within the framework of substructural logics (see [3]).

The first aim of this work is to prove a categorical equivalence between the category of NPc-lattices and its morphisms and a category whose objects are pairs of Brouwerian algebras and certain filters that we call regular. For this we follow the ideas of Odintsov in [4].

We then define Gödel NPc-lattices as those NPc-lattices with a prelinear negative cone, which turn to be equivalent to pairs of Gödel hoops and regular filters. For the case of finite Gödel NPc-lattices, by the duality of finite Gödel hoops and finite trees (see [1]) we also obtain a dual category, which consists in pairs of finite trees and certain subtrees.

Therefore we have a duality between a category of algebras and a category of combinatorics, which we use to characterize the free algebras in the variety of Gödel NPc-lattices.

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On an operation with regular elements

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In a previous work (see [1]) we have studied, in the context of residuated lattices, the operation B given by the greatest Boolean below a given element. In particular, our results hold for the class $\mathbb{ML}$ of meet-complemented lattices. Given an $\mathbf{M} \in \mathbb{ML}$ and $a \in M$, a is said to be *Boolean* iff there is an element $b \in M$ such that $a \wedge b = 0$ and $a \vee b = 1$, where, as usual, we use $\wedge$, $\vee$, 0, and 1 for the infimum, the supremum, the bottom, and the top of $\mathbf{M}$, respectively. The given definition is easily seen to be equivalent to saying that a is Boolean iff $a \vee \neg a = 1$. Accordingly, given an $\mathbf{M} \in \mathbb{ML}$, we have postulated the existence of an operation B such that, for all $a \in M$,

$$Ba = \max\{b \in M : b \leq a \text{ and } b \vee \neg b = 1\},$$

using the notation $\mathbb{ML}^B$ for the corresponding class. It is equivalent to postulate that there exists an operation B such that the following hold:

(BE1) $Bx \preccurlyeq x$,
(BE2) $Bx \vee \neg Bx \approx 1$, and
(BI) if $y \leq x$ & $y \vee \neg y \approx 1 \Rightarrow y \preccurlyeq Bx$,

where, resembling the terminology used in a Natural Deduction setting, the notations I and E come from Introduction and Elimination, respectively.

Now we deal with the operation that results when substituting the notion of regular for the notion of Boolean. Accordingly, given an $\mathbf{M} \in \mathbb{MSL}$, that stands for the class of meet-complemented meet-semilattices, we may postulate the existence of an operation R such that, for all $a \in M$,

$$Ra = \max\{b \in M : b \leq a \text{ and } \neg\neg b = b\},$$

using the notation $\mathbb{MSL}^R$ for the corresponding class.

Note that the minimum regular above is not a new operation, as it equals $\neg\neg$. On the other hand, operation R seems to deserve inspection.

We prove that R is the right adjoint of double meet-negation. We also prove that the resulting class is an equational class satisfying the Stone equality, i.e. $\neg x \vee \neg\neg x \approx 1$. Finally, it is also the case that the class of distributive meet-negated lattices with the greatest regular below is the same as the class of Stone distributive meet-negated lattices with the greatest Boolean below.

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Semi-intuitionistic logic with strong negation

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There is a well known interplay between the study of algebraic varieties and propositional calculus of various logics. Prime examples of this are boolean algebras and classical logic, and Heyting algebras and intuitionistic logic. After the class of Heyting algebras was generalized to the semi-Heyting algebras by H. Sankappanavar in [San08], its logic counterpart was developed by one of us in [Cor11] and further studied in [CV15].

Nelson algebras, or N-lattices were defined by H. Rasiowa [Ras58] to provide an algebraic semantics to the constructive logic with strong negation proposed by Nelson in [Nel49]. D. Vakarelov in [Vak77] presented a construction of Nelson algebras staring from Heyting ones. Applying this construction to semi-Heyting algebras, we obtained in [CV16] the variety of semi-Nelson algebras as a natural generalization of Nelson algebras. In this variety, the lattice of congruences of an algebra is determined through some of its deductive systems. Furthermore, the class of semi-Nelson algebras is arithmetical, has equationally definable principal congruences and has the congruence extension property.

It is the purpose of this work to present a Hilbert-style propositional calculus which is complete with respect to the algebras in this variety. Naming this logic *semi-intuitionistic logic with strong negation* was a natural choice. We believe that this logic will be of interest from the point of view of Many-Valued Logic, since its algebraic semantics show that it can provide many different interpretations for the implication connective. For example, on a chain with five elements, ten different semi-Nelson algebras may be defined, by changing the implication operation.

We present the algebraic motivation, defining semi-Nelson and semi-Heyting algebras. We then introduce the axioms and inference rule for the semi-intuitionistic logic with strong negation, together with some of their consequences. Finally we deal with completeness of the logic with respect to the class of semi-Nelson algebras, and present an axiomatic extension that has the variety of Nelson algebras as its algebraic semantics.

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A categorical equivalence for Nelson algebras

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The construction given by Kalman of Kleene algebras starting from distributive lattices, and by Vakarelov of Nelson algebras up from Heyting ones is generalized to obtain De Morgan algebras from distributive lattices. Necessary and sufficient conditions for these De Morgan algebras to be Nelson algebras are shown, and a characterization of the join-irreducible elements in the finite case is given. This construction is then used to establish an equivalence between the category of Nelson algebras and a category consisting of pairs of Heyting algebras and one of their filters.

Synonymy and categories of logics

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The study of categories of logics is motivated, among other reasons, by questions such as how to combine logics and when they are equivalent (see for example [Arn15] and [Cal07] and the bibliography therein). Categories of logics do not always have good categorical properties such as the existence of finite limits and colimits. One natural way to obtain from these categories new ones with better properties is to consider the quotient category induced by the interdemonstrability relation, as done in [MaM16].

Following the aforementioned works, we consider the category whose objects are Tarskian logics and whose morphisms are flexible translations that preserve the synonymy relation between formulas, in the sense of [Smi62]. This relation induces a congruence on the class of morphisms of this category.

In this communication, we study the quotient category induced by synonymy and some of its categorical properties.

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On the logic that preserves degrees of truth associated to involutive Stone algebras

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Let A be a De Morgan algebra. Denote by $K(A)$ the set of all elements $a \in A$ such that the De Morgan negation of a, $\neg a$, coincides with the complement of a. For every $a \in A$, let $K_a = \{k \in K(A) : a \leq k\}$ and, if K_a has a least element, denote by ∇a the least element of K_a. The class of all De Morgan algebras A such that for every $a \in A$, ∇a exists, and the map

$a \mapsto \nabla a$ is a lattice-homomorphism is called the class of involutive Stone algebras, denoted by **S**. These algebras were introduced by Cignoli and Sagastume ([1]) in connection to the theory of n-valued Łukasiewicz–Moisil algebras.

In this work, we focus on the logic that preserves degrees of truth $\mathbb{L}_{\mathbf{S}}^{\leq}$ associated to involutive Stone algebras. This follows a very general pattern that can be considered for any class of truth structure endowed with an ordering relation; and which intend to exploit manyvaluedness focussing on the notion of inference that results from preserving lower bounds of truth values, and hence not only preserving the value 1 (see [3, 4, 5]).

Among other things, we prove that $\mathbb{L}_{\mathbf{S}}^{\leq}$ is a manyvalued logic (with six truth values) that can be determined by a finite number of matrices (four matrices). Besides, we show that $\mathbb{L}_{\mathbf{S}}^{\leq}$ is a paraconsistent logic, moreover, we prove that it is a genuine LFI (Logic of Formal Inconsistence, [2]) with a consistence operator that can be defined in terms of the original set of connectives. Finally, we study the proof theory of $\mathbb{L}_{\mathbf{S}}^{\leq}$ providing a Gentzen calculus for it, which is sound and complete with respect to the logic.

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Una dualidad para semirretículos distributivos monótonos

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Las lógicas modales monótonas basadas en la lógica clásica o en la lógica intuicionista son generalizaciones de la lógica modal normal (clásica o intuicionista) en las cuales el axioma $m(\varphi \to \psi) \to (m\varphi \to m\psi)$ se debilita a una condición de monotonía, que puede ser expresada como un axioma $m(\varphi \wedge \psi) \to m\varphi$ o una regla (de $\varphi \to \psi$ se deriva $m\varphi \to m\psi$). En el trabajo que presentaremos hemos estudiado una dualidad estilo Stone para los $\wedge$-semirretículos distributivos con último elemento (en adelante semirretículos) dotados de un operador modal monótono. Los semirretículos se encuentran presentes en diversas estructuras algebraicas relacionadas al estudio de las lógicas no clásicas, por ejemplo en la semántica algebraica del fragmento $\{\to, \wedge, \top\}$ de la lógica intuicionista que es la variedad de los semirretículos implicativos.

Las extensiones canónicas fueron introducidas por Jonsson y Tarski para estudiar las algebras de Boole con operadores. El propósito principal era identificar qué forma debía tener el dual de una operación adicional. La teoría de las extensiones canónicas ha sido simplificada y generalizada para ser aplicada en estructuras algebraicas mas allá de las álgebras de Boole. En nuestro trabajo usamos la extensión canónica como una herramienta algebraica para representar los operadores modales por medio de relaciones en el espacio dual. La mayoría de los resultados de este trabajo son aplicables, bajo menores modificaciones, al estudio de los retículos distributivos acotados, semirretículos implicativos, álgebras de Heyting y álgebras de Boole con operadores monótonos.

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Congruencias que preservan aniquiladores en DN-álgebras

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Una *DN-álgebra* es un supremo-semirretículo con último elemento el cual cada filtro principal es un retículo distributivo acotado. Las DN-álgebras forman una variedad y generalizan tanto a las álgebras de Tarski como a los retículos distributivos acotados. Varios autores han estudiado estas estructuras en [5], [6] y [4]. Recientemente en [2], se desarrolla una dualidad topológica estilo Stone para la clase de las DN-álgebras. Como aplicación, dicha dualidad nos permite traducir y resolver en términos topológicos problemas algebraicos. Un buen ejemplo es la caracterización de las congruencias de una DN-álgebra a través de *N-subespacios* del espacio asociado a la DN-álgebra.

Por otro lado, en [7], se introduce la noción de *congruencia que preserva aniquiladores*, o *AP-congruencia*, en un retículo distributivo acotado A como una congruencia θ tal que para todo $a,b \in A$, si $a \wedge b \equiv_\theta 0$ entonces existe $c \in A$ tal que $a \wedge c = 0$ y $c \equiv_\theta b$. En [3] se obtienen nuevas equivalencias y una interpretación topológica de esta noción. Siguiendo los resultados desarrollados en [2] y [1], el objetivo de esta comunicación es extender el concepto de AP-congruencia a la clase de las DN-álgebras. Luego, representamos a las AP-congruencias a través de N-subespacios cumpliendo una condición adicional.

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Selfextensional logics with a nearlattice term

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The aim of this communication is to propose a definition of when a ternary term m of an algebraic language $\mathscr{L}$ is a distributive nearlattice term (DN-term for short) for a sentential logic §. Then, we show that selfextensional logics with a distributive nearlattice term m can be characterized as the sentential logics § for which there exists a class of algebras K such that the $\{m\}$-reducts of the algebras of K are distributive nearlattices and the consequence relation of § can be defined using the order induced by the term m on the algebras of K.

The notion of distributive nearlattice can be defined in two equivalent way. An algebra $\langle A, m\rangle$ of type (3) is a distributive nearlattice if satisfies some identities; and a ternary algebra $\langle A, m\rangle$ is a distributive nearlattice if and only if A with the binary term $x \vee y := m(x,x,y)$ is a join-semilattice such that for every $a \in A$, the upset $[a)$ is a distributive lattice with respect to the order induced by $\vee$.

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L_n^2-álgebras

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Teniendo en cuenta que la variedad $\mathscr{L}_n^2$ está constituida por las álgebras de Łukasiewicz m-generalizadas de orden n considerando $m = 2$. Es decir, las L_n^2-álgebras en las cuales $f^4x = x$. Mostramos propiedades de los átomos que serán de gran utilidad para describir detalladamente a las álgebras simples de $\mathscr{L}_n^2$. Más precisamente, hallamos el número de L_n^2-álgebras simples y su cardinal.

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A topological duality for tense $\mathbf{LM}_n$-algebras and applications

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In 2007, tense n-valued Łukasiewicz–Moisil algebras (or tense LM_n-algebras) were introduced by Diaconescu and Georgescu as an algebraic counterpart of the tense n-valued Moisil logic. In this paper we continue the study of the tense LM_n-algebras initiated by Figallo and Pelaitay in [1]. More precisely, we determine a topological duality for these algebras. This duality enable us not only to describe the tense LM_n-congruences on a tense LM_n-algebra, but also to characterize the simple and subdirectly irreducible tense LM_n-algebras. Furthermore, by means of the aforementioned duality a representation theorem for tense LM_n-algebras is proved, which was formulated and proved by a different method by Georgescu and Diaconescu in [2].

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Reductos implicativos de las MV-álgebras n-valuadas temporales

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En este trabajo se propone una definición para los $\{\to,1\}$-reductos de las MV-álgebras n-valuadas temporales descriptas como en [4].

Recordemos que una MV-álgebra *en el lenguaje de las álgebras de Wajsberg* es un álgebra $\langle A,\to,\sim,1\rangle$ de tipo $(2,1,0)$ si se satisfacen las siguientes ecuaciones:

(I1) $1\to x = x$
(I2) $(x\to y)\to((y\to z)\to(x\to z)) = 1$
(I3) $(x\to y)\to y = (y\to x)\to x$
(W) $(\sim x\to\sim y)\to(y\to x) = 1$

Por otro lado una terna (A,G,H) es una MV-*álgebra temporal* si A es una MV-álgebra y G, H son operadores unarios sobre A que satisfacen las siguientes propiedades:

(T1) $G(1) = 1, H(1) = 1,$
(T2) $G(x\to y)\le G(x)\to G(y), H(x\to y)\le H(x)\to H(y),$
(T3) $x\le GP(x), x\le HF(x)$, donde $P(x) =\sim H(\sim x)$ y $F(x) =\sim G(\sim x)$,
(T4) $F(x)\to G(y)\le G(x\to y), P(x)\to H(y)\le H(x\to y),$
(T5) $G(\sim x\to x)\le F(\sim x)\to G(x), H(\sim x\to x)\le P(\sim x)\to H(x),$
(T6) $G(\sim x)\to F(x)\le F(\sim x\to x), H(\sim x)\to P(x)\le P(\sim x\to x).$

Un reducto implicativo de una MV-álgebra n-valuada es un álgebra $\langle A,\to,1\rangle$ de tipo $(2,0)$ que satisface las identidades (I1) a (I3) y la propiedad adicional

(I4) $((x^n \to y) \to x) \to x = 1$,
where $x^0 \to y = y$, and $x^{(n+1)} \to y = x \to (x^n \to y)$ for all positive integer n.

En adelante para simplificar llamaremos I_{n+1}-álgebras a reductos implicativos de las MV-álgebras n-valuadas

Una terna (A, G, H) es una I_{n+1}-*álgebra temporal* o tI_{n+1}-*álgebra* si A es una I_{n+1}-álgebra y G, H son operadores unarios sobre A y tales que para todo $a \in A$ se satisfacen las siguientes propiedades:

(T1) $G_a(1) = 1, H_a(1) = 1$,
(T2) $G_a(x \to y) \leq G_a(x) \to G_a(y), H_a(x \to y) \leq H_a(x) \to H_a(y)$,
(T3) $x \leq G_a P_a(x), x \leq H_a F_a(x)$, donde $P_a(x) = \sim_a H_a(\sim_a x)$ y $F_a(x) = \sim_a G_a(\sim_a x)$,
(T4) $F_a(x) \to G_a(y) \leq G_a(x \to y), P_a(x) \to H_a(y) \leq H_a(x \to y)$,
(T5) $G_a(\sim_a x \to x) \leq F_a(\sim_a x) \to G_a(x), H_a(\sim_a x \to x) \leq P_a(\sim_a x) \to H_a(x)$,
(T6) $G_a(\sim_a x) \to F_a(x) \leq F_a(\sim_a x \to x), H_a(\sim_a x) \to P_a(x) \leq P_a(\sim_a x \to x)$,
where G_a y H_a son las correspondientes restricciones de G y H a $[a] = \{x \in A : a \leq x\}$, y $\sim_a x = x \to a$ respectivamente.

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Una dualidad topológica para álgebras de Boole bitopológicas

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Las álgebras de Boole topológicas, llamadas también “álgebras interiores” o “álgebras de clausura”, fueron estudiadas por Tarski [5] y Naturman [2]. Los espacios bitopológicos, a su vez, fueron estudiados por [1]. En [3], se presentan las álgebras de Boole bitopológicas, que relacionan y generalizan naturalmente las estructuras anteriormente mencionadas. Entre otras aplicaciones, permiten dar un enfoque topológico a la completación de Dedekind–MacNeille de álgebras de Heyting [4]. Definiremos explícitamente funtores que establecen una equivalencia natural entre las categorías de álgebras de Boole bitopológicas y las de ciertos espacios topológicos.

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Demiquantifiers on MV-algebras

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In this paper we introduce the notion of *demiquantifiers* on MV-algebras. Given an MV-algebra A, let $A^- = \{x \in A : x \leq \neg x\}$ and let $A^+ = \{x \in A : x \geq \neg x\}$. Demiquantifiers determine a new type of quantifiers on MV-algebras. These operators behave like an existential quantifier when restricted to the set A^- and like an universal quantifier when restricted to A^+ where A is an arbitrary MV-algebra. A particular and interesting case is constructed in the functional MV-algebra $[0,1]^X$ determined by all functions defined on a non-empty set X with values in the real unit interval $[0,1]$. In this case we define a demiquantifier $\exists_{\frac{1}{2}}$ having the following semantic interpretation: given a propositional function $f : X \to [0,1]$, then $\exists_{\frac{1}{2}} f = \frac{1}{2}$ if and only if $f^{-1}(\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon) \neq \emptyset$ for every $\varepsilon > 0$. In particular, when the image of f is a finite subset of $[0,1]$, then $\exists_{\frac{1}{2}} f = \frac{1}{2}$ if and only if $f(x_0) = \frac{1}{2}$ for some $x_0 \in X$. Taking into account that the classical existential quantifier $\exists$ satisfies $\exists f = 1$ if and only if $f^{-1}(1-\varepsilon, 1] \neq \emptyset$ for every $\varepsilon > 0$ and the universal quantifier $\forall$ satisfies $\forall f = 0$ if and only if $f^{-1}[0,\varepsilon) \neq \emptyset$ for every $\varepsilon > 0$, then the demiquantifier $\exists_{\frac{1}{2}}$ can be considered as a non-classical quantifier satisfying the corresponding property for the *neighborhoods* of the constant $\frac{1}{2}$. We prove that these quantifiers determine a variety and are interdefinable with the usual existential quantifiers on MV-algebras given by DiNola and Grigolia ([1]) provided the corresponding MV-algebras have a fixed point. Moreover, demiquantifiers generalize the notion of existenctial middle quantifier considered in [2] for the class of three-valued Łukasiewicz algebras.

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Semantics of triples for the first-order paraconsistent logic QCiore

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In this talk the logic **QCiore** is introduced as a first-order version of the 3-valued paraconsistent proposicional logic **LFI2**, introduced in [2] and additionally studied in [1] under the name of **Ciore**. As semantical counterpart for **QCiore** we consider the so-called **LFI2**-structures, which are defined as $\langle \mathfrak{A}, \|\cdot\|_{\mathbf{LFI2}} \rangle$, such that $\mathfrak{A}$ is a partial structure as introduced in [3], and $\|\cdot\|_{\mathbf{LFI2}}$ is a mapping which assigns to each formula of the language of **QCiore** $\mathscr{M}$-fuzzy set (a concept taken from [4]) over the set of all the variable assignments in the domain of $\mathfrak{A}$, where $\mathscr{M}$ is the logical matrix of **Ciore**.

Soundness and Completeness theorems for **QCiore** with respect to **LFI2**-structures are obtained, and a first study of **LFI2**-structures from the point of view of Model theory is developed.

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A model-theoretic study of the class of F-structures for mbC

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As it is well-known, many paraconsistent logic in the class of the *Logics of Formal Inconsistency* (in short LFIs, see [4]) do not allow us an algebrization by means of Blok-Pigozzi's method. However, some of them can be semantically characterized by non-deterministic structures such as Nmatrices and possible-translations semantics. A semantic of Nmatrices based on an special kind of multialgebra called *swap structures* was proposed in [2, 3] and an algebraic study of them was developed in [7].

The decidability of da Costa's calculi C_n (an special class of LFIs) was proved for the first time by M. Fidel in [8] by means of a novel algebraic-relational class of structures called *C_n-structures*. A C_n-structure is a triple $\langle \mathscr{A}, \{N_a\}_{a\in A}, \{N_a^{(n)}\}_{a\in A}\rangle$ such that $\mathscr{A}$ is a Boolean algebra with domain A and each N_a and $N_a^{(n)}$ is a non-empty subset of A. The intuitive meaning of $b \in N_a$ and $c \in N_a^{(n)}$ is that b and c are possible values for the paraconsistent negation $\neg a$ of a and for the well-behavior a° of a, respectively. This kind of structure was baptized by S. Odintsov as *Fidel structures* or **F**-structures. In [2, 3], a semantics of **F**-structures was found for **mbC** and several of its extensions.

In this talk, an initial study of the class of **F**-structures for **mbC** (for short, mbC-structures) from the point of view of Model Theory is proposed. From this perspective, the **F**-structures are seen as first-order structures over the signature of Boolean algebras expanded with two binary predicate symbols N and O for the paraconsistent negation and the consistency connective, respectively. As a consequence of this, notions and tools from Model Theory and Category Theory can be applied. This point of view allows us to consider notions such as substructures, union of chains, direct products, direct limits, congruences, quotient structures, ultraproducts, etc. In this sense, an result due to X. Caicedo ([1]) will be used. In that paper, conditions under which it is possible to have a Birkhoff-like theorem about subdirectly decomposition for first-order structures of classical model Theory are provided. Using this, a representation theorem for mbC-structures as subdirect poducts of subdirectly irreducible (s.i.) mbC-structures is obtained by adapting Caicedo's results. In particular, given an mbC-structure over the two-atoms Boolean algebra we can determine whether it is s.i. or not. Besides, every mbC-structure over the two-element Boolean algebra is proved to be subdirectly irreducible.

Finally, another decomposition theorem is obtained by using the notions of *weak substructure* and *weak isomorphism* by adapting results due to Fidel for C_n-structures. In this case, the mbC-structures over the two-element Boolean algebra play the role of s.i. structures.

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Definibilidad por fórmulas abiertas en estructuras relacionales

Pablo Ventura

En esta charla hablaremos sobre el siguiente problema computacional:

OpenDef:: Toma como entrada una estructura finita **A** y una relación T sobre el universo de **A**, y determina si hay una fórmula abierta tal que su extensión coincida con T.

Mostraremos que su complejidad es coNP-complete y luego presentarmos un algoritmo que toma ventaja de una caracterización semántica de la definibilidad abierta. Finalizaremos mostrando una implementación del algoritmo y su uso con algunos ejemplos.

Epistemic BL-algebras: An algebraic semantics for BL-possibilistic logic

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Possibilistic logic [4, 2] is a well-known uncertainty logic to reasoning with graded beliefs on classical propositions by means of necessity and possibility measures. When trying to extend the possibilistic belief model beyond the classical framework of Boolean propositions to many-valued propositions, one has to come up with appropriate extensions of the notion of necessity and possibility measures for them. In the particular context of BL-fuzzy logic [3], a natural generalization that we will consider is the following: given a BL-algebra $\mathscr{A}$, a possibility distribution $\pi : \Omega \mapsto \mathscr{A}$ on the set Ω of $\mathscr{A}$-propositional interpretations we define the following generalized possibility and necessity measures over $\mathscr{A}$-propositions:

$$\Pi(\varphi) = \sup_{w \in \Omega} \{\pi(w) * w(\varphi)\}, \quad N(\varphi) = \inf_{w \in \Omega} \{\pi(w) \to w(\varphi)\}$$

where $*, \to$ are the product and the implication in $\mathscr{A}$. In this setting, W is a non-empty set of worlds, $\pi : W \mapsto \mathscr{A}$ is a possibility distribution on W, and $e : Var \times W \mapsto \mathscr{A}$ is a $\mathscr{A}$-propositional evaluation in each world. The set of valid formulas in the class $\Pi\mathscr{A}$ of *possibilistic Kripke models*, is denoted by $Val(\Pi\mathscr{A})$. Finding an axiomatic characterization of $Val(\Pi\mathscr{A})$ is an open problem proposed by Hájek in Chapter 8 of [3].

With the intend to solve this open problem by introducing the variety of *Epistemic BL-algebras* as BL-algebras endowed with two operators $\forall$ and $\exists$, considering that the models resulting from the Kripke semantics are EBL-algebras. In this talk we are going to introduce EBL-algebras, we will show some examples and properties of them. We will also compare them with monadic BL-algebras as defined in [3].

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Relation between BL-possilistic logic and epistemic BL-algebras

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Possibilistic logic [2] is a well-known uncertainty logic to reasoning with graded beliefs on classical propositions by means of necessity and possiblity measures. From a logical point of view, possibilistic logic can be seen as a graded extension of the well-know modal logic of belief KD45. When we go beyond the classical framework of Boolean algebras of events to BL-algebras frameworks, one has to come up with appropriate extensions of the notion of necessity and possibility measures for BL-valued events. In current work, we consider the general problem of giving an axiomatization of BL-possibilistic logic. The particular case when BL is a Gödel algebra was solved in [1]. However in the general setting of BL-algebras that it is an open problem proposed by Hájek in Chapter 8 of [3]. In our presentation, this problem is addressed and solved for the particular case of finite BL-algebras. For that, we introduce a connection between BL-possibilistic models and epistemic BL-algebras which is showed to be an algebriac semantics for BL-possibilistic logic.

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$\mathfrak{F}$-multipliers and the localization of semi-Heyting algebras

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The class $\mathscr{SH}$ of semi-Heyting algebras was first considered by H. P. Sankappanavar in [7], and mainly studied by J. M. Cornejo in [1, 2, 4, 3]. These algebras represent a generalization of Heyting algebras. Nevertheless the behavior of semi-Heyting algebras is much more complicated than that of Heyting algebras.

The aim of this paper is to generalize some of the results established in [5], using the model of bounded distributive lattices from [6] to semi-Heyting algebras. To this end, we introduce the notion of ideal on semi-Heyting algebras, dual to that of filter, which allows us to consider a topology on them. Besides, we define the concept of $\mathfrak{F}$-multiplier, where $\mathfrak{F}$ is a topology on an semi-Heyting algebra L, which is used to construct the localization semi-Heyting algebra $L_{\mathfrak{F}}$. Furthermore, we prove that the semi-Heyting algebra of fractions L_S associated with an $\wedge$-closed system S of L is an semi-Heyting algebra of localization. In the last part of this paper we give an explicit description of the semi-Heyting algebras $L_{\mathfrak{F}}$ and L_S in the finite case.

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On Kalman's functor for bounded hemiimplicative semilattices and hemiimplicative lattices

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A hemiimplicative semilattice is an algebra $(H, \wedge, \to, 1)$ of type $(2,2,0)$ such that $(H, \wedge)$ is a meet semilattice, 1 is the greatest element with respect to the order, $a \to a = 1$ for every $a \in H$ and for every a, b, $c \in H$, if $a \leq b \to c$ then $a \wedge b \leq c$. A bounded hemiimplicative semilattice is an algebra $(H, \wedge, \to, 0, 1)$ of type $(2,2,0,0)$ such that $(H, \wedge, \to, 1)$ is a hemiimplicative semilattice and 0 is the first element with respect to the order. A hemiimplicative lattice is an algebra $(H, \wedge, \vee, \to, 0, 1)$ of type $(2,2,2,0,0)$ such that $(H, \wedge, \vee, 0, 1)$ is a bounded distributive lattice and $(H, \wedge, \to, 1)$ is a hemiimplicative semilattice.

In this work we study an equivalence for the categories of bounded hemiimplicative semilattices and hemiimplicative lattices, respectively, which is motivated by an old construction due J. Kalman that relates bounded distributive lattices and Kleene algebras.

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A canonical representation for free left-dioids

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Dioids (or *cubical monoids*) were introduced by Marco Grandis [1] in an attempt to decategorify the notion of cubical monad. These algebras can be relaxed in such a way that only left or right absorptions are assumed. Concretely, a *left-dioid* is an algebra $\mathbf{D} = \langle D, \oplus, \otimes, 0, 1\rangle$ that satisfies the conditions:

(D1) $\langle D, \oplus, 0\rangle$ is a monoid

(D2) $\langle D, \otimes, 1\rangle$ is a monoid

(D3) $0 \otimes x \approx 0$
(D4) $1 \oplus x \approx 1$

Motivated by categorical interpretations of functional programs, we found left-dioids related to computational effects in the presence of exceptions. In this work we introduce a canonical representation for the free left-dioid over a set, obtaining an implementation which can be presented in a language with a simple type system.

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Comunicaciones de Matemática Aplicada

Generación de ciclos en redes neuronales competitivas

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Consideramos redes neuronales recurrentes modeladas por medio de las llamadas redes lineales por sectores [1, 3]. Más específicamente, en este tipo de modelos cada neurona está representada por una ecuación diferencial ordinaria con una no linealidad con umbral (descripta por una función que varía linealmente para valores por encima de cero y se anula en caso contrario). Cada variable representa el nivel de actividad (*firing rate*) de la neurona correspondiente. El sistema de ecuaciones diferenciales resultante es continuo y no diferenciable.

La topología de la red y los impulsos externos que recibe cada neurona determinan la existencia de distintos atractores entre los que se observan, por ejemplo, equilibrios, ciclos límite y soluciones cuasi-periódicas [1, 3, 4]. El comportamiento oscilatorio y la multiestabilidad que emerge en este tipo de redes las convierte en modelos adecuados para el estudio de la codificación y recuperación de patrones de memoria [1].

En este trabajo comentaremos algunos resultados que obtuvimos en redes competitivas, es decir, redes en las que la interacción entre neuronas es siempre inhibitoria aunque no necesariamente simétrica. Consideramos los impulsos externos constantes como parámetros y utilizamos herramientas de la teoría de bifurcaciones de sistemas no diferenciables [2]. Determinamos condiciones bajo las cuales existen ciclos límite estables de la red generados a partir de bifurcaciones de equilibrios.

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Una metodología para el estudio de la frecuencia de ciclos límite en ecuaciones diferenciales con retardo

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En este trabajo se consideran sistemas de ecuaciones diferenciales con retardo [3], del tipo

$$x_1'(t) = f_1(x_1(t), x_2(t), x_1(t-\tau), x_2(t-\tau), \mu)$$
$$x_2'(t) = f_2(x_1(t), x_2(t), x_1(t-\tau), x_2(t-\tau), \mu).$$

Suponiendo que este sistema tiene un ciclo límite, se quiere estudiar la dependencia de la frecuencia de dicho ciclo con el parámetro μ. En particular interesa saber si esta es constante. La metodología desarrollada consiste en considerar a la familia de ciclos límite como función del tiempo y del parámetro, hacer un cambio de coordenadas en el tiempo y derivar las ecuaciones correspondientes respecto de μ. Se genera entonces un sistema de ecuaciones diferenciales lineales no autónomas con retardo para las variaciones de las coordenadas respecto de μ (sistema auxiliar).

Se puede ver que la familia de ciclos límite es isocrónica, es decir, su frecuencia no depende del parámetro, si y solo si el sistema lineal construido anteriormente tiene soluciones periódicas de período 2π.

En este trabajo las soluciones del sistema auxiliar se han obtenido numéricamente, sin embargo podrían aplicarse métodos analíticos para dicho fin. Se muestran varios ejemplos de familias iscrónicas y no isocrónicas, ver también [1, 2].

Se ha observado que en ciertos casos, como los mostrados en [2] existen infinitos ciclos de período 2π del sistema auxiliar. Este fenómeno presenta interés en sí mismo y se ha analizado con detalle.

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What kind of bonus point system makes the rugby teams more offensive?

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Using simple tools of game theory in a rigorous way, we compare the level of "offensiveness" that rugby teams have under different kind of punctuation systems usually used in some important tournaments around the world. Under the light of a static model, we will provide predictions about the behaviour of the teams, analyzing if they become more offensive when a bonus point is awarded for scoring more than 4 tries, or the winning team scores three tries more than the other team, or no bonus is awarded. Finally, using a dynamic model and some results from dynamic games (Masso–Neme), we will show what bonus system offers better payoffs for the involved teams.

Obstáculos esenciales para grafos arco-circulares Helly

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Un grafo arco-circular Helly es el grafo de intersección de un conjunto de arcos en un círculo que satisface la propiedad de Helly. Introducimos la noción de obstáculos esenciales, que es un refinamiento de la noción de obstáculos [1], y probamos que los obstáculos esenciales son precisamente los subgrafos arco-circulares inducidos prohibidos minimales para la clase de los grafos arco-circulares Helly. Mostramos que es posible encontrar en tiempo lineal, en cualquier obstáculo dado, algún subgrafo inducido prohibido minimal para la clase de los grafos arco-circulares Helly contenido como subgrafo inducido. Más aún, apoyándonos en un algoritmo de tiempo lineal existente para encontrar obstáculos inducidos en grafos arco-circulares [1], concluimos que es posible encontrar en tiempo lineal un obstáculo esencial inducido en cualquier grafo arco-circular que no es un grafo arco-circular Helly. El problema de encontrar una caracterización por subgrafos inducidos prohibidos, no restringida solo a grafos arco-circulares, para los grafos arco-circulares Helly permanece irresuelto. Como una respuesta parcial a este problema, hallamos la caracterización por subgrafos inducidos prohibidos minimales para la clase de los grafos arco-circulares Helly restringida a grafos que no contienen claw ni 5-wheel como subgrafo inducido. Más aún, mostramos que existe un algoritmo de tiempo lineal para encontrar, en cualquier grafo dado que no sea arco-circular Helly, un subgrafo inducido isomorfo a claw, a 5-wheel o a un subgrafo inducido prohibido minimal para la clase de los grafos arco-circulares Helly.

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