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Relevant Information and Relevant Logic

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Abstract

In this paper we define and develop an algebraic structure associated with the concept of *systems of relevant information* (**SRI**), which is a variant of the semilattice semantics proposed by A. Urquhart in [6]. The propositional relevant logic RP is introduced as the syntactical counterpart of the **SRI**-structures. This logic has a primitive-recursive decision procedure. The idea behind this logic is that of "Relevant Deduction", in which each premise is a block of information relevant to the conclusion. Finally, we prove that the class of **SRI**-structures is a sound and complete semantics for the logic RP.

Introduction

Relevant Logic was initially developed by A. Anderson and N. Belnap in axiomatic terms throughout their systems **R** and **E** (cf. [1]). There exists several semantical approaches to these logics (cf. [5]). As pointed in [4], however, there is no natural semantics for these systems.

We propose here the semantics of *systems of relevant information* (**SRI**), based on the semilattice semantics, introduced by A. Urquhart for the systems **R** and **E**. The underlying idea is that we can only infer a formula α from a state of information which is relevant to α. (From the classical point of view, a state of information is merely a set Γ of formulae from which we deduce the formula α.) We redefine this semantics throughout the **SRI**-structures, formed by states of information and introduce the relation $\preccurlyeq$ which works associated to the following idea: one state of information X is *logically stronger* than another state Y (written $X \preccurlyeq Y$) if, in the intended interpretation, there exists one formula α (associated to a basic block of information in Y) which can be (classically) derived from the set Γ of formulae associated to the state X.

Additionally, the relation of *relevance* $\Vdash$ is defined as follows: $X \Vdash \alpha$ means that the state of information X is relevant for obtaining the formula α. This semantics corresponds intuitively to the idea of minimal set of premises needed to infer a formula α.

On the other hand, another important problem related to the axiomatic approach for the relevant logic is the decision problem. It was proved in the literature that several fragments of the systems **E** and **R** (and other related systems) are decidable. It was finally proven that neither **R** nor **E** have decision procedures (cf. [2]).

In the present approach to Relevant Logic, having in mind the central problem of finding a decidable Relevant Logic motivated by pragmatic reasons associated to automated deduction in Artificial Intelligence, we are primarily interested in investigating a decidable logic with relevant characteristics, namely RP, which functions as a sort of *relevant filter* of the classical logic. This logic, based on [3], is associated with the idea of relevant deduction. So, the system RP is a subsystem of the classical logic which accepts only the classical deductions and where, in intuitive terms, there are no unnecessary premises. So, valid formulae in RP are classically valid formulae which are additionally relevantly deduced from the empty set of premises.

We prove that the class of **SRI**-structures is sound and complete for the logic RP. We thus conclude that there exist decidable Relevant Logics with "natural" semantics in the sense proposed by Pogorzelski in [4].

1 Systems of Relevant Information

In this section we introduce the central concept of the paper: Information systems endowed with a relation of relevance.

Let us suppose that we have a formal language $\mathbb{L}$ for knowledge representation. For the sake of simplicity, we will assume that $\mathbb{L}$ is a (classical) propositional language defined over the signature

$$\Sigma = \{p_i \mid i \in \mathbb{N}\} \cup \{\neg, \rightarrow, \wedge, \vee\}.$$

From now on, $\mathfrak{L}_C$ will denote the classical propositional logic over the language $\mathbb{L}$, and $\models_C$ will stand for the (semantical) consequence relation associated to $\mathfrak{L}_C$.

In order to define an information system $\mathcal{S}$ over $\mathbb{L}$ we will briefly analyse the main characteristics that we intend to have in $\mathcal{S}$:

- The elements of $\mathcal{S}$ will be called *states of information* (from now on **s.i.**).
- One s.i. X may contain more information than another s.i. Y; in this case, we write $Y \leq X$; the relation $\leq$ should be a partial order.
- Given X and Y, we can define the minimum s.i which contains more information than X and Y (the *supremum* or *concatenation of* X *and* Y).
- Given X and Y, we can define the maximum s.i which contains less information than X and Y (the *infimum* or *common information of* X *and* Y).
- There exists the null s.i. $\mathbf{0}$.

• Given X and Y, there exists the maximum state Z with less information than X and without common information with Y (the *complement of* Y *in* X).

• There exists atomic pieces of information, called *blocks of information* (from now on **b.i**), corresponding to the formulae in $\mathbb{L}$.

• Each X is obtained as a finite concatenation of b.i.'s.

• There exists a relation $\preccurlyeq$ such that $X \preccurlyeq Y$ means that the s.i. X is *logically stronger* than the s.i. Y.

Formally, we define the following:

Definition 1.1 A *System of Relevant Information* (**SRI**) is a structure $\mathcal{S} = \langle |\mathcal{S}|, \leq, \mathbf{0}, \preccurlyeq \rangle$ such that:

(i) $\langle |\mathcal{S}|, \leq, \mathbf{0} \rangle$ is a distributive lattice with minimum $\mathbf{0}$; elements in $|\mathcal{S}|$ are *states of information* (**s.i.**).

(ii) For all $X, Y \in |\mathcal{S}|$ there exists $X - Y = Max\{Z \leq X \mid Z \sqcap Y = \mathbf{0}\}$.

(iii) For all $\alpha \in \mathbb{L}$ there exists a s.i. X_α such that $\alpha \equiv \beta$ (in $\mathfrak{L}_C$) iff $X_\alpha = X_\beta$, and $\alpha \not\equiv \beta$ implies that $X_\alpha \not\leq X_\beta$. Each X_α is called a *block of information* (**b.i.**).

(iv) For each s.i. X there exists b.i. $X_{\alpha_1}, \ldots, X_{\alpha_n}$ such that

$$X = X_{\alpha_1} \sqcup \cdots \sqcup X_{\alpha_n}.$$

(v) The relation $\preccurlyeq \subseteq |\mathcal{S}| \times |\mathcal{S}|$ satisfies: $X \preccurlyeq X_\alpha$ iff, for every finite sequence $X_{\alpha_1}, \ldots, X_{\alpha_n}$ of b.i.'s, if $X = X_{\alpha_1} \sqcup \cdots \sqcup X_{\alpha_n}$ then $\alpha_1, \ldots, \alpha_n \models_C \alpha$. Furthermore $X \preccurlyeq X_{\alpha_1} \sqcup \cdots \sqcup X_{\alpha_n}$ iff there exists $1 \leq i \leq n$ such that $X \preccurlyeq X_{\alpha_i}$. ■

It is easy to see that the decomposition $X = X_{\alpha_1} \sqcup \cdots \sqcup X_{\alpha_n}$ is unique up to (classical) logical equivalence, and each b.i. is an atom, that is: $X \leq X_\alpha$ implies that $X = X_\alpha$ or $X = \mathbf{0}$. Therefore the relation $\preccurlyeq$ is well defined. On the other hand, if X and Y are given by

$$X = X_{\alpha_1} \sqcup \cdots \sqcup X_{\alpha_n} \sqcup X_{\beta_1} \sqcup \cdots \sqcup X_{\beta_m},$$

$$Y = X_{\beta_1} \sqcup \cdots \sqcup X_{\beta_m} \sqcup X_{\gamma_1} \sqcup \cdots \sqcup X_{\gamma_k}$$

then $X - Y = X_{\alpha_1} \sqcup \cdots \sqcup X_{\alpha_n}$.

A **SRI** induces a relevance relation as follows:

Definition 1.2 Let $\mathcal{S}$ be a **SRI**. Given a s.i. X and a formula α, we say that X is *relevant* for α, denoted $X \Vdash \alpha$, if the following conditions hold:

(1) if α is a literal then $X \Vdash \alpha$ iff $X \preccurlyeq X_\alpha$ and $Y \not\preccurlyeq X_\alpha$ for all $Y < X$ (here, $Y < X$ means that $Y \leq X$ and $Y \neq X$);
(2) $X \Vdash \alpha \to \beta$ iff $X \sqcup X_\alpha \Vdash \beta$ and $X \sqcup X_{\neg\beta} \Vdash \neg\alpha$;
(3) $X \Vdash \alpha \vee \beta$ iff $X \sqcup X_{\neg\alpha} \Vdash \beta$ and $X \sqcup X_{\neg\beta} \Vdash \alpha$;
(4) $X \sqcup Y \Vdash \alpha \wedge \beta$ iff $X \Vdash \alpha$, $Y \Vdash \beta$, $X \not\preccurlyeq Y - X$ and $Y \not\preccurlyeq X - Y$;
(5) $X \sqcup Y \Vdash \neg(\alpha \to \beta)$ iff $X \Vdash \alpha$, $Y \Vdash \neg\beta$, $X \not\preccurlyeq Y - X$ and $Y \not\preccurlyeq X - Y$;
(6) $X \sqcup Y \Vdash \neg(\alpha \vee \beta)$ iff $X \Vdash \neg\alpha$, $Y \Vdash \neg\beta$, $X \not\preccurlyeq Y - X$ and $Y \not\preccurlyeq X - Y$;
(7) $X \Vdash \neg(\alpha \wedge \beta)$ iff $X \sqcup X_\alpha \Vdash \neg\beta$ and $X \sqcup X_\beta \Vdash \neg\alpha$;
(8) $X \Vdash \neg\neg\alpha$ iff $X \Vdash \alpha$. ■

Is immediate to see that the formula $\alpha \vee \beta$ is regarded as equivalent to $(\neg\alpha \rightarrow \beta) \wedge (\neg\beta \rightarrow \alpha)$. This eliminates asymmetries, as we will see on the next sections.

It is interesting to note that a state of information X is constructed as a finite concatenation of blocks. Since each block is associated to a formula, X is associated to a finite set of formulae representing the informational content of X. Observe that the content of X is not merely given by the associated set of formulae, but the number of blocks defining X is also considered. Then, the state of information $X_\alpha \sqcup X_\beta$ is different from the state of information $X_{\alpha\wedge\beta}$.

2 The relevant system RW

We begin now our study of the logical (syntactical) counterpart of the Systems of Relevant Information. In this section we will study a simple *relevant* consequence relation, called RW, introduced in [3].

Definition 2.1 Given a set $\Gamma \cup \{\alpha\}$ of propositional formulae, we say that $\Gamma \models_{RW} \alpha$ (*α is a consequence of Γ in the logic RW*) if the following is true:

(1) $\Gamma \models_C \alpha$, and

(2) for all $\beta \in \Gamma$, $\Gamma - \{\beta\} \not\models_C \alpha$. ■

For example: $\alpha, \alpha \rightarrow \beta \models_{RW} \beta$ but $\alpha, \alpha \rightarrow \beta \not\models_{RW} \alpha$. An obvious consequence of Definition 2.1 is the following:

Lemma 2.2 Let $\Gamma \cup \{\alpha, \beta\}$ be a set of formulae. If $\Gamma \models_{RW} \alpha$ and $\models_C \alpha \leftrightarrow \beta$, then $\Gamma \models_{RW} \beta$. □

A number of properties of RW can be found in [3], for example:

$\Gamma, \alpha \models_{RW} \beta$ implies that $\Gamma \models_{RW} \alpha \rightarrow \beta$

(deduction theorem). The converse is not true in general:

$\alpha \models_{RW} \beta \rightarrow \alpha$ but $\alpha, \beta \not\models_{RW} \alpha$.

In RW the principle $\alpha, \beta \models_{RW} \alpha \wedge \beta$ is not valid in general: by Lemma 2.2, it is immediate that

$$\alpha, \alpha \wedge \beta \not\models_{RW} \alpha \wedge (\alpha \wedge \beta).$$

On the other hand, RW does not satisfy the **CUT** rule:

$$\Gamma \models_{RW} \alpha, \quad \alpha, \Delta \models_{RW} \beta \quad \text{implies that} \quad \Gamma, \Delta \models_{RW} \beta.$$

In fact,

$$\{p, p \rightarrow q\} \models_{RW} q, \quad q, \{q \rightarrow p\} \models_{RW} p \quad \text{but} \quad \{p, p \rightarrow q, q \rightarrow p\} \not\models_{RW} p.$$

This example suggest the following:

Proposition 2.3 Let C_{RW} be the consequence operator of RW, and let Γ be a set of formulae. If $C_{RW}(\Gamma) \neq \emptyset$ then there exists no formulae $\gamma, \beta \in \Gamma$, $\gamma \neq \beta$, such that $\gamma \models_C \beta$.

Proof: Let $\alpha \in C_{RW}(\Gamma)$ and suppose that $\gamma, \beta \in \Gamma$, $\gamma \neq \beta$, such that $\gamma \models_C \beta$. Since $\Gamma \models_{RW} \alpha$ we obtain $\Gamma \models_C \alpha$. On the other hand, $\gamma \models_C \beta$ implies $\Gamma - \{\beta\} \models_C \alpha$, a contradiction. □

Corollary 2.4 Let Γ be a set of formulae. If there exists formulae $\gamma, \beta \in \Gamma$, $\gamma \neq \beta$, such that $\gamma \models_C \beta$, then $C_{RW}(\Gamma) = \emptyset$. □

Corollary 2.5 If $C_{RW}(\Gamma) \neq \emptyset$, then $C_{RW}(C_{RW}(\Gamma)) = \emptyset$.

Proof: Let $\Delta = C_{RW}(\Gamma) \neq \emptyset$, and let $\alpha \in \Delta$. Then $\Gamma \models_{RW} \alpha$ and $\Gamma \models_{RW} \neg\neg\alpha$. Since $\alpha, \neg\neg\alpha \in \Delta$ and $\alpha \models_C \neg\neg\alpha$ we infer, by Corollary 2.4, that $C_{RW}(\Delta) = C_{RW}(C_{RW}(\Gamma)) = \emptyset$. □

Corollary 2.6 The consequence operator C_{RW} is cyclic.

Proof: Let Γ be a set of formulae. If $\Gamma = \emptyset$ then we obtain, by succesive applications of the operator C_{RW}, the following cyclic sequence:

$$\emptyset, T, \emptyset, T, ...,$$

where $T = C_{RW}(\emptyset)$ is the set of tautologies of RW (= set of tautologies of the classical logic $\mathfrak{L}_C$). If $\emptyset \neq \Gamma \neq C_{RW}(\Delta)$ for all Δ, we obtain, by succesive applications of the operator C_{RW}, the following cyclic sequence:

$$\Gamma, C_{RW}(\Gamma), \emptyset, T, \emptyset, T, ...$$

Finally, if $\emptyset \neq \Gamma = C_{RW}(\Delta)$ for some Δ then we obtain, by succesive applications of the operator C_{RW}, the following cyclic sequence:

$$\Gamma, \emptyset, T, \emptyset, T, ... \quad \square$$

As a direct consequence of Definition 2.1 we have that the tautologies of RW coincide with the clasical ones. In order to circumvent this difficulty, a modification of the logic RW was proposed in [3], so as to obtain a system that we call $RW1$. This system is defined by restricting the derivability of formulae of the form $\alpha \rightarrow \beta$ *exactly* to the cases determined by the converse of the deduction theorem, that is:

$$\Gamma, \alpha \models_{RW1} \beta \quad \text{iff} \quad \Gamma \models_{RW1} \alpha \rightarrow \beta.$$

The logic $RW1$ has its difficulties, however: for example

$\neg\alpha \models_{RW1} \alpha \rightarrow \beta$ but $\neg\alpha \not\models_{RW1} \neg\beta \rightarrow \neg\alpha$.

This shows that $RW1$ only works in the implicative fragment of RW.

3 The system RP

Finally we are ready to introduce our main logical system, the logic RP. It is obtained as a refinement of the logic RW, and it is defined recursively.

Definition 3.1 The logic RP is defined as follows: let $\Gamma \cup \{\alpha, \beta\}$ be a set of formulae; then

(1) $\Gamma \models_{RP} \alpha$	iff	$\Gamma \models_{RW} \alpha$ (if α is a literal);
(2) $\Gamma \models_{RP} \alpha \to \beta$	iff	$\Gamma, \alpha \models_{RP} \beta$ and $\Gamma, \neg\beta \models_{RP} \neg\alpha$;
(3) $\Gamma \models_{RP} \alpha \vee \beta$	iff	$\Gamma, \neg\alpha \models_{RP} \beta$ and $\Gamma, \neg\beta \models_{RP} \alpha$;
(4) $\Gamma \cup \Delta \models_{RP} \alpha \wedge \beta$	iff	$\Gamma \models_{RP} \alpha$, $\Delta \models_{RP} \beta$, $\Gamma \not\models_C \gamma$ for all $\gamma \in \Delta - \Gamma$ and $\Delta \not\models_C \gamma$ for all $\gamma \in \Gamma - \Delta$;
(5) $\Gamma \cup \Delta \models_{RP} \neg(\alpha \to \beta)$	iff	$\Gamma \models_{RP} \alpha$, $\Delta \models_{RP} \neg\beta$, $\Gamma \not\models_C \gamma$ for all $\gamma \in \Delta - \Gamma$ and $\Delta \not\models_C \gamma$ for all $\gamma \in \Gamma - \Delta$;
(6) $\Gamma \cup \Delta \models_{RP} \neg(\alpha \vee \beta)$	iff	$\Gamma \models_{RP} \neg\alpha$, $\Delta \models_{RP} \neg\beta$, $\Gamma \not\models_C \gamma$ for all $\gamma \in \Delta - \Gamma$ and $\Delta \not\models_C \gamma$ for all $\gamma \in \Gamma - \Delta$;
(7) $\Gamma \models_{RP} \neg(\alpha \wedge \beta)$	iff	$\Gamma, \alpha \models_{RP} \neg\beta$ and $\Gamma, \beta \models_{RP} \neg\alpha$;
(8) $\Gamma \models_{RP} \neg\neg\alpha$	iff	$\Gamma \models_{RP} \alpha$. ■

Is RP a relevant logic in the sense stated in the Introduction? The next proposition gives a positive answer to this question.

Proposition 3.2 Let $\Gamma \cup \{\alpha\}$ be a set of formulae. Then

$$\Gamma \models_{RP} \alpha \quad \text{implies that} \quad \Gamma \models_{RW} \alpha.$$

Proof: By an easy induction on the complexity of α. □

Remarks 3.3 (i) It is clear that the converse of Proposition 3.2 is not true:

$$\alpha \models_{RW} \beta \to \alpha \quad \text{but} \quad \alpha \not\models_{RP} \beta \to \alpha$$

because $\alpha, \beta \not\models_{RW} \alpha$.
(ii) Clause (3) in Definition 3.1 means that formulae of the form $\alpha \vee \beta$ are considered as $(\neg\alpha \to \beta) \wedge (\neg\beta \to \alpha)$. This eliminates asymmetries: let's consider the alternative clause

$$(3)' \ \Gamma \models_{RP} \alpha \vee \beta \quad \text{iff} \quad \Gamma, \neg\alpha \models_{RP} \beta$$

in the place of (3). This implies that $p \models_{RP} p \vee q$ (because $p, \neg p \models_{RP} q$) but $q \not\models_{RP} p \vee q$ (because $q, \neg p \not\models_{RP} q$). On the other hand, clause (3) guarantees a desirable symmetrical behavior of $\models_{RP}$: $p \not\models_{RP} p \vee q$ and $q \not\models_{RP} p \vee q$.
(iii) Analogously, if we substitute (2) in Definition 3.1 by the alternative clause

$$(2)' \ \Gamma \models_{RP} \alpha \to \beta \quad \text{iff} \quad \Gamma, \alpha \models_{RP} \beta$$

we obtain: $\neg p \models_{RP} p \to q$ but $q \not\models_{RP} p \to q$, as long as clause (2) produces

$$\neg p \not\models_{RP} p \to q \quad \text{and} \quad q \not\models_{RP} p \to q.$$

This is an interesting result from the relevantist point of view: the falsehood of p does not imply that p is relevant to every q. In the same vein, from q we would not infer that any p is relevant for q. ■

4 Substitutions in RP

We briefly analyse the effect of substitutions in the logic RP. We will prove that changing formulae by (classically) equivalent ones on the left-hand side of $\models_{RP}$ have no effects, but the right-hand side of $\models_{RP}$ is susceptible to substitutions.

Proposition 4.1 Let $\Gamma \cup \{\alpha\}$ be a set of formulae. For each $\beta \in \Gamma$ consider a formula β' such that β' is (classically) equivalent to β, and let

$$\Gamma' = \{\beta' \mid \beta \in \Gamma\}.$$

Then $\Gamma \models_{RP} \alpha$ implies that $\Gamma' \models_{RP} \alpha$.

Proof: Immediate, by induction on the complexity of α. □

Corollary 4.2 (Deduction theorem) Let $\Gamma \cup \{\alpha, \beta\}$ be a set of formulae. Then

$$\Gamma \models_{RP} \alpha \rightarrow \beta \text{ iff } \Gamma, \alpha \models_{RP} \beta \text{ and } \Gamma, \neg\beta \models_{RP} \neg\alpha \text{ (iff } \Gamma \models_{RP} \neg\beta \rightarrow \neg\alpha).$$

Proof: Straightforward. □

On the other hand, Proposition 4.1 is not longer true if we consider the right-hand side of the relation $\models_{RP}$:

$$p \models_{RP} p \quad \text{but} \quad p \not\models_{RP} p \wedge (p \vee q).$$

Moreover, in general we cannot substitute a propositional variable for another formula in a deduction. Let's denote by $\alpha[^{\beta}/_{p}]$ the formula obtained from α by the substitution of every occurrence of a propositional variable p in α by a formula β. If Γ is a set of formulae, then we define

$$\Gamma[^{\beta}/_{p}] = \{\gamma[^{\beta}/_{p}] \mid \gamma \in \Gamma\}.$$

Proposition 4.3 Let $\Gamma \cup \{\alpha\}$ be a set of formulae, p a propositional variable occurring in α, and let q be a new propositional variable. Then $\Gamma \models_{RP} \alpha$ implies that $\Gamma[^{p\wedge(p\vee q)}/_{p}] \not\models_{RP} \alpha[^{p\wedge(p\vee q)}/_{p}]$.

Proof: Since p is (classically) equivalent to $p \wedge (p \vee q)$, then β is (classically) equivalent to $\beta[^{p\wedge(p\vee q)}/_{p}]$ for all $\beta \in \Gamma$. If $\Gamma \models_{RP} \alpha$ then, by Proposition 4.1, we infer that $\Gamma[^{p\wedge(p\vee q)}/_{p}] \models_{RP} \alpha$. The rest of the proof is an easy induction on the complexity of α. □

5 Completeness of RP

In this section we will prove that the relevant logic RP corresponds to the syntactical counterpart of the **SRI**-structures. That is, the inferences in the context of systems of relevant information correspond exactly to the deductions in the logic RP. This shows that RP has a natural semantics given by the class of **SRI**-structures.

Theorem 5.1 (Soundness) Let $\Gamma = \{\alpha_1, ..., \alpha_n\}$ be a finite set of formulae, and let α be a formula. Then:

$$\text{if} \quad X_{\alpha_1} \sqcup \cdots \sqcup X_{\alpha_n} \Vdash \alpha \quad \text{in every } \mathbf{SRI}\text{-structure } \mathcal{S} \quad \text{then} \quad \Gamma \models_{RP} \alpha.$$

In particular,

$$\text{if} \quad 0 \Vdash \alpha \quad \text{in every } \mathbf{SRI}\text{-structure } \mathcal{S} \quad \text{then} \quad \models_{RP} \alpha.$$

Proof: Let $\{\alpha_1, ..., \alpha_n, \alpha\}$ be a finite set of formulae such that, for every **SRI**-structure, $X_{\alpha_1} \sqcup \cdots \sqcup X_{\alpha_n} \Vdash \alpha$. Let's define a canonical **SRI**-structure $\mathcal{S}$ as follows: let

$$\mathbf{L_C} = \{[\beta] \mid \beta \in \mathbb{L}\}$$

be the Lindembaum algebra of $\mathfrak{L}_C$, where $[\beta] = \{\gamma \in \mathbb{L} \mid \beta \equiv \gamma \text{ in } \mathfrak{L}_C\}$ is the equivalence class of the formula $\beta \in \mathbb{L}$ in $\mathbf{L_C}$. Define $|\mathcal{S}| = \mathcal{P}f(\mathbf{L_C})$, the set of finite subsets of $\mathbf{L_C}$. For each $\beta \in \mathbb{L}$ set $X_\beta = \{[\beta]\}$. Then, the lattice $\langle |\mathcal{S}|, \subseteq, \emptyset\rangle$ satisfies conditions (i)-(iv) of Definition 1.1. Finally, set

$$\{[\gamma_1], ..., [\gamma_k]\} \preccurlyeq \{[\beta_1], ..., [\beta_m]\}$$

iff there exists $1 \leq i \leq m$ such that $\gamma_1, ..., \gamma_k \models_C \beta_i$. It is easy to see that condition (v) of Definition 1.1 is also satisfied, so $\mathcal{S} = \langle |\mathcal{S}|, \subseteq, \emptyset, \preccurlyeq\rangle$ is a **SRI**-structure. By hypothesis, $X_{\alpha_1} \sqcup \cdots \sqcup X_{\alpha_n} \Vdash \alpha$ in $\mathcal{S}$. A straightforward induction on the complexity of α shows that $\{\alpha_1, ..., \alpha_n\} \models_{RP} \alpha$. □

Theorem 5.2 (Completeness) Let $\Gamma = \{\alpha_1, ..., \alpha_n\}$ be a finite set of formulae, and let α be a formula. Then

$$\text{if } \Gamma \models_{RP} \alpha \quad \text{then} \quad X_{\alpha_1} \sqcup \cdots \sqcup X_{\alpha_n} \Vdash \alpha \quad \text{in every } \mathbf{SRI}\text{-structure } \mathcal{S}.$$

In particular,

$$\text{if } \models_{RP} \alpha \quad \text{then} \quad \mathbf{0} \Vdash \alpha \quad \text{in every } \mathbf{SRI}\text{-structure } \mathcal{S}.$$

Proof: Let $\{\alpha_1, ..., \alpha_n, \alpha\}$ be a finite set of formulae such that $\alpha_1, ..., \alpha_n \models_{RP} \alpha$, and let $\mathcal{S}$ be a **SRI**-structure. By induction on the complexity of α we will prove that $X_{\alpha_1} \sqcup \cdots \sqcup X_{\alpha_n} \Vdash \alpha$. If α is a literal, then $\alpha_1, ..., \alpha_n \models_{RW} \alpha$, thus it is immediate from the definitions above that $X_{\alpha_1} \sqcup \cdots \sqcup X_{\alpha_n} \Vdash \alpha$. Assume that the result is valid for every formula α with complexity $\leq k$ $(k \geq 0)$, and consider α a (non-literal) formula with complexity $k+1$.
Case 1: $\alpha = \beta \rightarrow \gamma$. Then $\alpha_1, ..., \alpha_n \models_{RP} \alpha$ implies that $\alpha_1, ..., \alpha_n, \beta \models_{RP} \gamma$ and $\alpha_1, ..., \alpha_n, \neg\gamma \models_{RP} \neg\beta$ and thus, by induction hypothesis,

$$X_{\alpha_1} \sqcup \cdots \sqcup X_{\alpha_n} \sqcup X_\beta \Vdash \gamma \quad \text{and} \quad X_{\alpha_1} \sqcup \cdots \sqcup X_{\alpha_n} \sqcup X_{\neg\gamma} \Vdash \neg\beta.$$

The cases $\alpha = \beta \vee \gamma$ and $\alpha = \neg(\beta \wedge \gamma)$ are proven analogously.
Case 2: $\alpha = \beta \wedge \gamma$. Then $\{\beta_1, ..., \beta_r\} \cup \{\gamma_1, ..., \gamma_s\} \models_{RP} \alpha$ implies that:

(i) $\{\beta_1, ..., \beta_r\} \models_{RP} \beta$;
(ii) $\{\gamma_1, ..., \gamma_s\} \models_{RP} \gamma$;
(iii) $\{\beta_1, ..., \beta_r\} \not\models_C \delta$ for every $\delta \in \{\gamma_1, ..., \gamma_s\} - \{\beta_1, ..., \beta_r\}$; and
(iv) $\{\gamma_1, ..., \gamma_s\} \not\models_C \delta$ for every $\delta \in \{\beta_1, ..., \beta_r\} - \{\gamma_1, ..., \gamma_s\}$.

By induction hypothesis and the definitions above it is straightforward to see that $X_{\beta_1} \sqcup \cdots \sqcup X_{\beta_r} \sqcup X_{\gamma_1} \sqcup \cdots \sqcup X_{\gamma_s} \Vdash \alpha$. The cases $\alpha = \neg(\beta \rightarrow \gamma)$ and $\alpha = \neg(\beta \vee \gamma)$ are treated similarly. The case $\alpha = \neg\neg\beta$ follows by induction hypothesis. □.

6 Concluding Remarks

We have introduced a decidable logic, based on relevance criteria. The natural semantics for this logic corresponds to systems of information equipped with a relevance relation. Note that, whereas the traditional approaches to relevance (as the systems **R** and **E**) use a new non-classical connective $\Rightarrow$ (relevant implication), in RP no new connectives are introduced. The idea of *relevance* in RP is just a metalinguistic one, related to the demands for a method for checking (classically) redundant information.

As questions deserving further investigation we can mention:

- to generalize this techniques to other non-classical decidable logics, in order to obtain a filtering of "relevant" formulae;
- to extend the usual notion of algebraic semantics in order to define structures over sets of formulae in the place of structures over formulae (as in the case of Lindembaum algebras). This possibility was already considered in [6].

7 References

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